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ARTICLE II.

*Account of a Trigonometrical Survey of Massachusetts, by Simeon Borden, Esq., with a Comparison of its Results with those obtained from Astronomical Observations, by Robert Treat Paine, Esq., communicated by Mr. Borden. Read 16th April, 1841.**

Before entering upon the details of the Massachusetts Survey, and Map, I propose to give you a few particulars respecting the history of its origin and progress.

SECTION I.—HISTORICAL PARTICULARS RELATIVE TO THE SURVEY OF MASSACHUSETTS.

The legislature of Massachusetts passed resolves on the first of March, 1830, requiring the City of Boston and the several towns in the Commonwealth to make an accurate map, each of its own territory, upon a scale of one hundred rods to an inch, and deposite the same with the Secretary of State. These resolves go much into detail upon the subject. On the third of the same month, a resolve was passed authorizing the Governor to appoint a skilful surveyor to make a survey of the state upon trigonometrical principles, combining astronomical observations therewith.

Some time in the following summer, the Governor appointed Robert Treat Paine, Esq., as principal Engineer, with James Stevens, Esq., of Newport, Rhode Island, for his principal assistant. During the latter part of the season Mr. Stevens visited Washington, and borrowed a number of instruments from the general government. He also called upon me to make him an apparatus for measuring a base line. In the fall he selected a location for a base line and partially traced it. This, I think, is all that was done in 1830. Mr. Paine, I believe, ordered some instruments which he received the following season. In the course of the ensuing winter I made the measuring apparatus, and made the necessary repairs in the instruments which Mr. Stevens had procured at Washington. Toward the latter end of April or first of May, 1831, Mr. Stevens had all his apparatus transported to the base line: and some time in June I joined him to assist in the measurement. Mr. Paine was considered at this time as the principal engineer; but as he had not taken any part in the field operations in which Mr. Stevens had been engaged, nor given him any directions, Governor Lincoln ordered Mr. Stevens to report directly to him, which

* It is proper to mention that this paper was not originally intended for the Transactions, but formed part of a correspondence between the author and Sears C. Walker, Esq.

he ever afterwards did; and thus, in effect, Mr. Paine became the astronomical, and Mr. Stevens the topographical surveyor.

Mr. Paine, I think, in the course of this season, commenced making astronomical and chronometrical observations.

When we had completed the measurement of the base line, I left the service; but I again engaged, as an assistant to Mr. Stevens, the following spring, and have been ever since connected with the survey. After Mr. Stevens had conducted the topographical survey about three years, he resigned his appointment. As I had assisted him in every part of the work which had been performed, and of course was fully acquainted with all that had been done, the governor thought it proper to place the survey under my charge. The field work of the survey I completed in the spring of 1838; and I presumed that when I had made the necessary trigonometrical computations, my labours would end; or that, if I were continued, it would be in the capacity of an assistant to Mr. Paine, since I supposed that Mr. Paine would at this point take charge of the construction of the map, and particularly of the mathematical calculations necessary to render the work complete. In this, however, I was mistaken, for Mr. Paine had made his final report a short time before the triangulation was in readiness for the commencement of compiling the map; and thus the responsibility of completing the work unexpectedly devolved upon me.

After I had completed the field work, and had calculated a sufficient number of the main triangles to cover a section of fifty miles square of the western portion of the state, I commenced the work of compiling the map, when I found the town maps which had been returned to the Secretary so incorrectly drawn as to render it impossible, in their actual state, to make a satisfactory map from them. I was then obliged to go into the field again, with four or five assistants, and make corrections; and this operation has been one of continued perplexity, and has cost the state, in my department alone, at the least estimation, ten thousand dollars more than it would have done, had the towns executed their portion of the work in good faith. It is my opinion that had the work been performed from beginning to end under the direction of a faithful and competent engineer, it might have been executed, at the rate of compensation which has been paid, for many thousand dollars less than it has now cost.

Still the survey of the state of Massachusetts, including eight thousand two hundred and thirty square miles of territory, and having an indented sea coast of about three hundred miles, has been completed, in little more than ten years, at an expense of only sixty-one thousand, three hundred and twenty-two dollars. A brief and imperfect account of this work is offered in the present paper. Of its merits and the manner in which we have executed the trust committed to our charge, we leave others to judge.

SECTION II.—OF THE STANDARD SCALE AND APPARATUS FOR MEASURING THE BASE.

The standard of length first selected was a scale of two feet, constructed upon compensating principles, and of course unsuitable for subdivision. Being afterwards compared at Washington, by Mr. Hassler, Superintendent of the United States' Coast Survey, with his 82 inch scale of Troughton's construction, which is an exact copy from the well known Troughton scale of Sir George Shuckburgh, it was found to be 0.0018 inches too short, at the temperature of 57°.5 Fah. But a part of the triangles having been, previous

to this comparison, computed according to the Massachusetts scale, it was thought best to complete the calculations in the same manner, and make correction afterwards, when the proper standard should be fixed upon. For this standard, Hassler's 82 inch Troughton, at the temperature 62° Fahr., was chosen.

The apparatus with which the base was measured was constructed upon compensating principles, and was supported with a strong and firmly soldered tin tube between seven and eight inches in diameter, and, while in use, placed upon tressels properly contrived for carrying on such operations, a simple cross upon a plate of silver marking the terminus of the measure. The termination of each measure was observed with suitable compound microscopes, furnished with cross hairs, and supported upon stands conveniently contrived for such operations.

The base line chosen for the Massachusetts survey was on the Connecticut river, above Northampton, and was found to be 39009.73 feet, or 7.3882 miles long. In addition to the information concerning its location, I have concluded to furnish, in a tabulated form, a comparison of its two measurements; the line having been twice measured. The details are of course omitted. The termini of the base line were marked thus, ⊕, upon copper bolts of about three-fourths of an inch in diameter, which were driven firmly into holes drilled for the purpose in large stones, which were carefully imbedded in the earth about eighteen inches beneath the surface. The southern terminus of the base line is situated near the middle of a large field belonging to Josiah Ellis in the town of Hatfield, upon the west side of the Connecticut river. Its bearing and distance from the meeting-house in Hatfield village will probably be sufficiently well indicated by stating the latitude and longitude of each.

	Latitude.	Longitude.
Hatfield Church, - - - - -	42° 22' 10" .97	72° 36' 11" .73
Southern terminus of base line, - - - - -	42 22 21 .07	72 37 12 .67.

The northern terminus of the base line is situated in a lot of land upon the south bank of Bloody Brook, in the town of Deerfield. Its bearing and distance from the Bloody Brook meeting-house will be sufficiently well shown by comparing the latitudes and longitudes of each.

The latitude of Bloody Brook Church being 42° .29' .04" .82 North. [Longitude 72° .36' .40" .78 West.
 " " Northern terminus of base line, 42 .28 .46 .44 " " 72 .37 .05 .28 "

In measuring the base line we commenced from the north end; and at the end of every thousand feet we drove small plugs or piles of wood into the ground, and marked upon them, by means of a small brass wire driven into the top of each, the termination of every thousand feet, as nearly as we conveniently could, without consuming too much time. We also left marks at the terminus of each day's work. With such of these marks as we found undisturbed upon our second measurement, we compared our work.

I would observe, that in making up the list of those comparisons, where the first measurement exceeds the second, I have marked the differences thus, +; and where the second measurement exceeds the first I have marked the differences thus, —. The comparisons are shown in the following table.

Number of the comparison.	Length of first measure.	Length of second measure.	First difference.	Second difference.
1	11995.225	11995.450	— 0.225	.225
2	23994.307	23994.413	— 0.106	.119
3	35991.383	35991.772	— 0.389	.283
4	47991.350	47991.638	— 0.288	.101
5	59991.266	59991.554	— 0.288	.000
6	71991.325	71991.444	— 0.119	.169
7	95981.617	95981.886	— 0.269	.150
8	107981.560	107981.665	— 0.105	.164
9	119981.242	119981.552	— 0.310	.205
10	131980.062	131980.360	— 0.298	.012
11	143979.732	143980.046	— 0.314	.016
12	155974.794	155975.128	— 0.334	.020
13	167970.783	167971.008	— 0.225	.109
14	179962.639	179962.938	— 0.299	.074
15	203961.267	203961.493	— 0.226	.073
16	227960.220	227960.355	— 0.135	.091
17	251960.280	251960.300	— 0.020	.115
18	287960.554	287960.276	+ 0.278	.258
19	299960.660	299960.261	+ 0.399	.121
20	311960.715	311960.261	+ 0.454	.055
21	347959.994	347959.461	+ 0.533	.079
22	371959.892	371959.182	+ 0.710	.177
23	386337.443	386336.615	+ 0.828	.118
24	428915.406	428915.268	+ 0.138	.690
25	467914.561	467914.324	+ 0.237	.099
Total sum of all the differences, in inches,				3.523

I would here remark, that our apparatus having been constructed upon principles entirely new, so far as it respects an application to works of this kind, we found considerable trouble in our first attempts at measuring, and it was not until we had measured that portion of the line represented by the column marked "first measure," beginning at the bottom, or No. 25, and extending as far as Nos. 19, or 18, or thereabouts, that we had perfected all the adjustments of our apparatus. This portion of the line was, moreover, very uneven, or hilly, so that we were frequently obliged to measure over ridges upon inclinations of from 3 to $7\frac{1}{2}$ degrees. The combination of all these troubles and difficulties may account for the large errors apparent in that part of the measurement. Since we found less difficulty in measuring after we had perfected our apparatus, it is reasonable to suppose that the second column gives the most accurate measurement: but as the final results were so near alike, I have given equal weight to both.

The above table contains the length of that portion of the base line which was measured with the fifty feet apparatus, to which is to be added 265.466 inches, which were taken with the beam compass. Before making the addition, I would observe that the exact length of the fifty feet measure, as determined by the Massachusetts standard after the measurement of the line was completed, was found to be only 599.998 inches.

Therefore, a mean of the measures exhibited in the table,

reduced to Massachusetts standard, is equal to 467,912.882 inches.

Add for the measure taken with the beam compass, 265.466 "

Whole length of base line, 468,178.348

Which being further reduced to Mr. Hassler's standard at 62° of Fahrenheit, and to the level of the sea, gives the final adopted length at 39009.73 feet, or 7.388 miles.

The heights of the north and south ends of the base line, above the mean level of the sea, were determined in the same manner as those of the other stations to be mentioned hereafter. At the time of measuring the base, however, we found its north end to be 49.55 feet higher than the south by the following process, namely: For the purpose of measuring the line accurately and conveniently, we divided it into a series of hypothenuses upon which the measuring apparatus was carried; the inclination of these hypothenuses was determined by precise measures of the depression or elevation of each; and the difference in the height of their termini was then computed. The coincidence of the difference of level found by the two independent methods serves to test the precision of both. Thus the height of the north end of the base above the mean level of the sea was found by vertical triangulation to be 220.02 feet, and of the south end by a similar process 169.59 feet; the difference being 50.43 feet, which differs only 0.88 feet from that obtained by inclination of the hypothenuses.

For measuring the base line eight persons were employed. This number, which included the principal and assistants, was found necessary to perform the work dexterously. Three small tents were used for covering the microscopes when necessary.

SECT. III.—OF THE TRIANGULATION, AND THE PREPARATION OF THE MAP OF MASSACHUSETTS.

The instrument used in measuring horizontal angles was a twelve-inch repeating Theodolite, made by Troughton for Mr. Hassler at the commencement of the coast survey, and fully described in the second series of the Transactions of the American Philosophical Society, published at Philadelphia in 1825, page 328. I would refer to that paper for a description of it, and will merely add, that I found it necessary to supply the instrument with a firmer clamping apparatus, and with a more powerful telescope to be used in measuring the azimuth angles. This telescope was forty-six inches in focal length. It is now in the hands of Major Graham, upon the north-eastern boundary survey.

The vertical angles were measured with the small telescope and vertical circle which Mr. Troughton made for the instrument, it being difficult to attach the vertical circle to the arbour of the large telescope. The clamps of the vertical circle were also insufficient for repeating the measures of angles of much magnitude; but as our vertical angles were for the most part very small, I did not make any change or addition to its clamping apparatus.

After the work fell into my hands, our party never consisted, principal and assistants, of more than three persons, while we were employed in making the triangulation; with the exception, however, of occasionally employing a person a few hours to assist us in putting up a heavy signal, and sometimes for a day or so to assist in felling trees which obstructed our operations.

We had one circular tent, about nine feet in diameter at the upper end of the main body, which is about six and a half feet from the ground, and much larger at the lower end. The roof or top of this tent was in the form of an obtuse cone, and the sides or walls were of four separate curtains, with suitable strings for closing them when

necessary. This tent was employed to protect the theodolite, when in use, from the effects of the sun and wind, and at other times from the weather.

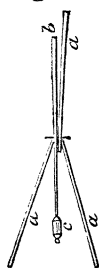
The state of Massachusetts is so densely settled, that we could without much inconvenience find lodgings with the inhabitants near almost all of our stations; so that we did not provide ourselves with camp equipage. When we found it necessary to encamp at a station, we built up small cabins, which we covered with brushwood and turf. We were only obliged to encamp regularly at three stations.

For the greater portion of the time, while we were triangulating the state, we were provided with two horses, and with a spring wagon made particularly for the service, and of rather large dimensions, for the convenience of stowing our instruments and baggage. While the survey was under the direction of Mr. Stevens, and after the base line was measured, an additional horse and a small buggy were provided for his accommodation.

The time occupied at a station, in measuring angles, varied from one day to twenty-five, depending much upon the state of the weather, and the amount of labour to be performed. I suppose that the average time spent at stations of much magnitude would be from five to ten days. Had we been provided with an instrument of suitable dimensions* and properly constructed, more than a third of the time occupied in measuring angles might have been saved.

At the commencement of the survey, it became an object of much interest, to contrive a signal which it would be convenient to erect in almost every situation imaginable; which would be firm and secure when erected, at least so far as regards high winds and storms; which would admit of being easily adjusted, repaired, or altered when found necessary; which would furnish a convenient support for our tent, and at the same time allow the theodolite, without an inconvenient derangement, to be placed directly over the centre of the station. The signal which we then contrived, and afterward used through the whole survey, is supposed to possess all of the properties named above, in a very satisfactory degree. The dimensions of the signals vary somewhat on account of the different condition or situation of the sites or stations over which they are erected, and not unfrequently have we been obliged to accommodate them to the size of the timber near at hand. The signal staffs were from fifteen feet to eighty feet in height. When we have had occasion to erect a very high signal, we have found it necessary, for the purpose of securing the top, which in these cases is the only portion used, from being swayed by the wind, to attach four slender wire guys or stays, which may be secured to pins driven into the ground for the purpose, or to trees or other objects, at hand. Lines would not make suitable stays, on account of their liability to lengthen and shorten with slight variations of moisture.

The following figures or sketches are intended to represent the signal in all its various attitudes or positions. Fig. 1 represents the signal as framed or put together upon the ground previous to erecting it;—a, a, a, represent the braces or legs which support the signal staff b, when it is erect; c represents a bulb or balloon, (as we frequently call it,) of common cotton or any other cheap white cloth, to render the signal more conspicuous under certain degrees and conditions of the light. It is formed by sewing together some pieces of cloth into the size or diameter of a flour barrel and about a yard and a half long. Two flour barrel hoops are then placed within the bag, equidistant from its end, and secured



* The instrument should be as large as can be conveniently transported.

there by sewing the bag firmly to them. Their distance apart should be from a half to three-fourths of a yard. The cloth thus prepared is firmly secured to the upper end of the signal staff, by puckering the ends of the bag, and sewing them to it with strong twine; taking care so to arrange the ends as to cause them to support the signal bulb or balloon concentrically with the signal staff. We have sometimes bored a hole into the top of the signal staff and inserted therein a tuft of pine or other dark-coloured boughs,—a dark-coloured signal being frequently more conspicuous than a light-coloured one.

The materials which compose the signal frame are frequently procured from the nearest forest trees; and are put together without any other dressing or preparation than merely depriving them of their branches, and fashioning their ends in a suitable manner to unite them into a firm and compact frame or tripod. The frame is secured by passing through the upper end of the braces and through a point near the middle of the signal staff a strong wooden pin or an iron bolt. The whole will be sufficiently apparent from an inspection of the figures, without further description. The signal thus described, so far as my experience goes, has been found to answer well in all situations. It is readily and easily adjusted upon bare smooth rocks,—that is, as bare and as smooth as they are ever found in their natural position. I have placed them in soft swampy lands, and have found them to stand in those situations a long time without any perceptible derangement.

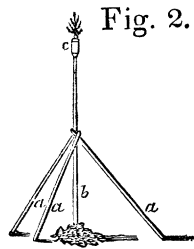


Fig. 2. Fig. 2. represents a signal erected, and in suitable adjustment to be observed from other stations for the purpose of measuring angles therewith.

I would here state that, as by far the greater portion of the primitive trigonometrical stations in Massachusetts are situated upon the tops of hills and mountains whose summits are bare rocks, or rocks very slightly covered, here and there, with earth, we have marked these points by drilling a small hole about two inches deep into the rocks, and inserting therein a small copper bolt marked thus, \otimes ; and after having put our signal frame together, with the aid of a small tackle,—a derrick staff (as it is called in the language of seamen) secured in its vertical position with guy ropes or stays,—the signal is readily erected over the bolt, and that part of the signal staff which extends upwards from the meeting of the braces adjusted perpendicularly over the centre of the bolt. For determining when the signal was in proper adjustment, we used two small telescopes mounted very similarly to a transit telescope. These instruments, when placed in such a manner that the vertical planes of their motion, when passing through the centre of the upper bolt, would intersect each other nearly at right angles, would readily show, when pointed to any part of the signal staff, whether it was or was not in proper adjustment. By a movement of the lower ends of the braces, which is very easily performed, the whole of the signal frame can be moved in any direction desired. Having put the main body of the signal frame in its proper situation, the upper end of the signal staff may be rendered plumb by moving the lower end in an opposite direction. The lower end of the signal staff comes very near the ground, but should not be allowed to touch it, and when it has obtained its proper place, the lower end is secured, and that of course secures also the upper end, by piling a heap of small stones about it, if they can be found near at hand; or, when they cannot, by throwing about it a small mound of earth. Many signals of the kind described, which I erected at the commencement of the survey, were standing two years

ago without any apparent derangement, and I presume will continue to stand and retain their adjustment until they decay. Before I dismiss this subject I would further mention, that whenever our stations happened to be situated where rocks were not to be found, as upon Cape Cod, Nantucket, &c., we marked the stations by burying a bottle about eighteen inches beneath the surface, first filling it with sand, so that it should not collapse if it should happen to be broken.

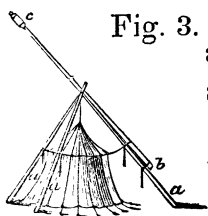


Fig. 3. Shows the position of the signal while we are measuring angles at the station. A sketch of the tent which protects the theodolite, is seen suspended from the frame.

Fig. 4. Shows the position of the signal when we put new cloth or a tuft of pine or other species of boughs upon it, or make any repairs, additions or alterations connected with the top of the signal staff.

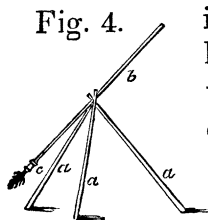


Fig. 4.

When the signal frames have been light or slender, we have bored holes into the lower ends of the braces and inserted strong pins, which we have loaded with stones to prevent the wind from blowing them down. And where stones were not to be found, we have driven strong pins into the earth, near the ends of the braces, and nearly at right angles therewith, which we have nailed firmly to the frame.

Respecting atmospheric phenomena, I would state that I have rarely been able to measure angles with a satisfactory degree of accuracy, whenever objects appeared in a striking degree looming up, as the sailors call it. These loomings or mirages are frequently seen, particularly upon the sea coast. I have often seen portions of the tops of mountains, valleys, trees, and houses double, that is, one above another. Sometimes I have seen objects triple, but I have never seen their images inverted. My experience shows that objects are refracted side-ways during extraordinary mirages; and I am very confident that they are frequently laterally refracted when there is no apparent mirage.

As respects the transparency of the atmosphere, or more correctly speaking that state of the atmosphere which permits distant objects to be distinctly seen, I have made many observations, with the view of ascertaining the reason why distant objects cannot be distinctly seen on certain days when the atmosphere appears to be uncommonly transparent. As I am not aware that any person has published an account of this apparent paradox, I will endeavour to state my experience upon this subject in as brief a manner as I well can. Hints of this kind may prompt others to investigate the subject; and whether my theory be found true or false, will be of little importance, provided the real cause be discovered, and we be enabled to predict correctly the state of the atmosphere, with respect to seeing distant objects distinctly. To the engineer engaged in trigonometrical operations this knowledge will be of great value, as it may save him the trouble of much useless and vexatious labour. It may also be useful at times to those engineers who have never been engaged in very extensive surveys, and of course can know but little from their own experience of the different states of the atmosphere; particularly when they have occasion to run a line of considerable extent, or wish to observe very distant objects. The curious traveller might also add much to his gratification, could he be

able to form a correct opinion of the condition of the atmosphere, favourable for viewing distant objects.

At the latter end of the month of November, 1835, I was at the trigonometrical station in Fairhaven endeavouring to measure the angles which were deemed necessary from that station. Soon after our arrival, we had a fall of snow which covered the ground several inches thick. Severely cold weather succeeded it; and although the atmosphere appeared uncommonly transparent to the unassisted eye, still we were unable to see our distant signals, particularly those upon Martha's Vineyard, and the Elizabeth Islands. The line of sight to these signals lay across the waters of Buzzard's Bay, and the Vineyard Sound. In a few days after the snow had fallen, the weather became warmer, and a rain ensued, which took off nearly all the snow. On the morning following the rain, namely, the fifth of December, we went early to the station, and discovered that all of our signals could be seen with tolerable distinctness, although the atmosphere was apparently not near so transparent as it had been at several of our previous visits to the station. The heavens were densely clouded. I think the sun did not appear at all, or if seen it must have been so very dimly as not to have perceptible influence upon the atmosphere. The day of course was, comparatively speaking, a dark one.

We commenced our operations of measuring angles as rapidly as we could; but we had not been long engaged in our work before the atmosphere began to present a very curdled appearance, and our signals to appear dim. In a short time they appeared plainer again, and then again more dim: the appearances thus alternating. Having a thermometer at hand, I directed Mr. Read, one of my assistants, to observe that instrument, and see if any changes took place in it corresponding with the changes in the appearance of the signals; and we soon ascertained that these changes always occurred together. The day was a very remarkable one for our operations, which we were enabled to prosecute without intermission until we had completed all we desired at the stations; and from the time we first commenced observing the thermometer until our work had ended, a slight change of the instrument could be easily detected from the appearance of the signals, particularly when looking across water. I think the change of appearance was not so strikingly apparent across the land. Having made these observations, I endeavoured to estimate the temperature of the surface of the land and water across which our observations were made. The sun not having shone at all, or at most not sufficiently bright to have any perceptible effect during the time of our observations, the surface of the earth might be considered of a uniform temperature. It was frozen, and the preceding night had been warm and rainy; and I therefore concluded that the surface of the earth must be at or near a freezing temperature, namely, 32° of Fah. The water, of course, must have been several degrees warmer. Still it was very cold; for a few days previous, we had had a considerable fall of snow, which, as is well known, chills water very rapidly. The weather which succeeded the snow storm had been extremely cold. I did not make any memoranda at the time, or if I did I cannot now find them, but I presume the thermometer must have stood as low as 10° or 12° , until within twelve or fifteen hours before the observations were made. The waters of Buzzard's Bay and the Vineyard Sound are very shallow, and ebb and flow very little; of course the waters could receive but a slight influence from the warm

water of the deep ocean in the course of a few hours.* The probability therefore is, that the surface of the water could not have been many degrees above freezing; and I suppose it must have been as low as 40° , if not lower. The thermometer at the station, I find, by my memoranda, stood generally at 34° , and did not vacillate more than two degrees. The surface of the earth, as before stated, is supposed to have been of the temperature of 32° . These numbers will show that there was a very near agreement between the temperature of the air, and the temperature of the surface across which the observations were made. I came therefore to the conclusion that the nearer the atmosphere and the surface of the earth or water across which the line of sight lies are to the same temperature, the more distinctly can distant objects be seen in an atmosphere of equal transparency. Since the morning on which these observations were made, I have observed many phenomena, natural and artificial, which might be cited in corroboration of the above conclusion; and I have seldom seen any thing to disprove it.

Before we finally dismiss this subject it may be well to describe, by way of elucidation, the different appearances of distant objects, commencing our description at a time when the objects appear plain and distinct, and continuing the description through all the different grades of appearances until they have entirely faded away, or until they cannot be longer seen.

Before entering upon the description, I would premise, that the atmosphere is supposed to be transparent; that is, free from fogs, smoke, dust, or other heterogeneous matter.

In choosing a time to commence our observations, I have selected the morning, as it is not unusual that all the phenomena which I shall notice succeed each in the order of their description. We therefore commence in the morning of a favourable day. Let us then suppose that when the sun has risen sufficiently to illuminate the distant objects, they are seen distinctly. We have been led to the conclusion, that when distant objects are distinctly seen, the atmosphere and the surface over which our line of sight passes must be of the same temperature; and experience has shown us that soon after the sun rises, objects are generally more distinctly seen than at any other time of the day, previous to within an hour or two of the sun's setting. And our reason will teach us that the surface of the earth and the atmosphere are oftener of the same temperature, morning and evening, than at other times of the day; therefore there must then be less of ascending and descending currents in the atmosphere.

The objects at this time of the day (morning) appear perfectly still, and present a well defined and distinct outline. I have of course supposed them to be stationary. As the sun rises, the surface of the earth absorbs warmth more readily than the air, and of course the stratum of air which lies in immediate contact with the earth becomes rarefied and rises, forming ascending currents while the vacuum is supplied by descending currents. In consequence of this condition of the atmosphere, distant objects* present a waving appearance;—the light reflected from them appears to be sluggishly refracted from right to left and from left to right,—the image of the objects appearing of about the same dimensions as when first observed, and with still a distinct outline. The sun continues to rise:—the heat increases, and with it ascending and descending currents in the atmosphere;—the images of the objects lose their wavy appearance, and appear larger and fainter;—

* In support of this conclusion I would state the fact, that the ocean frequently freezes in the Vineyard Sound and at Nantucket, so extensively, that no water can be seen for many successive days.

the refractions from right to left and from left to right succeed each other with such rapidity that all further appearance of distinct outline disappears, and in proportion as their apparent size increases, their distinctness diminishes until, if the objects be small and far off, no traces of them can be seen.

If we suppose a diminution of heat, from the temperature of distinct sight, in the same ratio that we have been supposing an increase, (which frequently takes place in the afternoon of a cold day,) objects will present similar appearances and vanish in the same manner.

In the foregoing remarks, I have supposed the atmosphere to possess, during the time of observation, the same degree of transparency.

Respecting the process of determining the azimuths of the stations, I would state that the system adopted in preparing the work for the map, is the same we use in all our calculations. I shall therefore describe it in connexion therewith.

First, we divided the state into sections of about fifty miles square each, and determined the direction of meridians through the most central trigonometrical station in each section. Two of these meridians were determined by direct astronomical observations, and the other meridians were calculated. Angles were carefully measured from the two observed meridians with all the most conspicuous signals, which were in sight from the stations, (which we shall call for the sake of distinction *a*,) through which the meridians pass. The direction of the sides of the triangles radiating from the *a* stations were determined from the angles measured directly with the meridians. The azimuths of the sides of the triangles radiating from the stations seen from stations *a*, (which we shall call *b*,) were predicated upon the angles measured at the stations *b*, from the signals standing upon stations *a*. The azimuths of the sides of the triangles radiating from the stations seen from station *b*, (which we shall call *c*,) were determined from the angles measured at station *c*, from the signals standing upon stations *b*; and in like manner the same system was continued until we reached the extreme station of the section, or any desired station within the limits of the survey. The same system was carried forward between the two observed meridians, making each meridional station alternately station *a*. The difference in the azimuth determined by computing from a series of angles, from one meridional station to the other, with that determined from astronomical observations, gives the inclination of the meridians.

I would here remark, that carrying forward the computed azimuths upon this system, supposes that the meridians of the stations *a*, *b*, *c*, &c., are all parallel. I would further remark, that the azimuths of the sides of the triangles determined from the stations *a*, remain unaltered, for the reason that they have been carefully determined directly from the meridian, which must be considered of the highest possible authority, and they may for that reason be called standard lines. A line or side of a triangle extending from one station *b*, to another station *b*, is considered differently. Thus the azimuth of a line or side of a triangle extending from station *b*, No. 1, to station *b*, No. 2, (using the numbers merely for distinction's sake,) should be exactly the reverse of each other; that is, if the azimuths of the line determined from station *b*, No. 1, to station *b*, No. 2, is South $40^{\circ} 10' 20''$ West, the azimuth of the same line, determined from station *b*, No. 2, should be North $40^{\circ} 10' 20''$ East. But we will suppose that the azimuths do not exactly agree, (as it frequently

* I would wish to have it understood that the objects selected are slender poles, spires of churches, or something of the kind, because the reasoning will apply more readily to objects of that form.

happens;) that the azimuth of station, *b* No. 2, determined from station *b*, No. 1, is South $40^{\circ} 10' 22''$ West, and the azimuth of station *b*, No. 1, determined from station *b*, No. 2 is North $40^{\circ} 10' 20''$ East. As these are not exact reverse courses, a mean course is taken, and applied in the calculations as the azimuth of the line. Thus: $40^{\circ} 10' 22'' + 40^{\circ} 10' 20'' = 80^{\circ} 20' 42''$ and $\frac{80^{\circ} .20' .42''}{2} = 40^{\circ} 10' 21''$, which is taken as the bearing or azimuth from *b*, No. 1, to *b*, No. 2, reversing the names of the points as before explained: hence station *b*, No. 2, bears from *b*, No. 1, South $40^{\circ} 10' 21''$ West, and *b*, No. 1, bears from *b*, No. 2, North $40^{\circ} 10' 21''$ East. The same principle has been carried through the whole work. Having thus determined the azimuthal bearings of one station from another throughout the whole of a section, (the distances also from one station to the other, having been previously calculated according to the method of Legendre,) we proceed to make upon the principles of plane trigonometry, a table of Northings, Southings, Eastings, and Westings, as the case may require, of each station in the section, or any other station in the survey to which we desire to know the bearing and distance from station *a*, of the section. The whole calculated and summed up from station *a*, through which, as before stated, the true meridian passes. The station *a*, we shall therefore call the zero point. Then, with the Northing or Southing, Easting or Westing, of a point to which our table has been carried, (which in fact constitutes two sides or legs of a right angled triangle,) we calculate on the principles of plane trigonometry, the angle at the zero point, which gives the bearing or azimuth of the distant station from that point. And in a similar manner, the azimuth of any point within the scope of the table, may be determined from said zero point. To determine an azimuth or bearing from any other station than the zero point, it becomes necessary to calculate a meridian for that station, and for that purpose we must know the value of the meridional and perpendicular degrees. The method of managing these calculations will become apparent, when we show the method of determining the value of a degree perpendicular to the meridian, from the inclinations or convergence of two observed meridians. It will therefore be unnecessary to go further into an explanation of this subject in this place.

For the purpose of testing the accuracy of the main triangulation, I have made several double computations of the latitude and longitude of points situated in different parts of the state. In the first place, I computed the latitude and longitude of our primitive station, upon French's Hill, in the town of Peru, and upon the top of the Green Mountain ridge. I then computed the latitude and longitude of Tuft's Hill, from the French's Hill station, and also from the State House, which, if the triangulation had been accurately performed, and no mistake made in our calculations, should have both given the same position to the Hill. The table shows a discrepancy in latitude of $0'' .05 = 5$ feet, and in longitude of $0'' .08 = 6$ feet. This is not a greater error than might arise, in calculations of this kind, by using tables carried to only seven places of decimals; and, as I possessed no better tables, I could not make the computations with greater accuracy. The latitudes and longitudes of the other points given in the table were calculated, in a similar manner, from the stations named against them; and as the object of the table cannot be misunderstood, it will be unnecessary to give further explanations.

Names of Stations.	Where situated.	Latitude.	Longitude.	Names of points from which the computations emanated.
French's Hill	Peru	42° 25' 30." 67.	73° 02' 39." 27.	State House.
Tuft's Hill	New-Braintree	42 19 01. 87 42 19 01. 92	72 05 25. 00 72 05 24. 92	From French's Hill in Peru. " State House.
Mount Tom	Northampton	42 14 30. 97 42 14 30. 92	72 39 17. 10 72 39 17. 06	" French's Hill in Peru. " Tuft's Hill, N. Braintree.
Wachusett Mount	Princeton	42 29 21. 23 42 29 21. 20	71 53 33. 88 71 53 33. 89	" State House. " French's Hill—Peru.
Highland Light	Cape Cod	42 02 23. 33 42 02 23. 39	70 03 55. 27 70 03 55. 43	" State House. " Hyannis—Barnstable.
Light House	Monomoy Point	41 33 34. 96 41 33 35. 00	69 59 56. 24 69 59 56. 24	" State House. " Hyannis—Barnstable.
South Tower	Nantucket	41 16 56. 59 41 16 56. 64	70 06 13. 91 70 06 13. 86	" State House. " Hyannis—Barnstable.
Court House	Barnstable	41 42 06. 08 41 42 06. 06	70 18 33. 80 70 18 33. 82	" State House. " Hyannis—Barnstable.
Wanomet Hill	Plymouth	41 55 38. 92 41 55 38. 85	70 35 44. 96 70 35 45. 03	" State House. " Hyannis—Barnstable.

It cannot be said that the table shows more than one unsatisfactory result, and that refers to Highland Light upon Cape Cod, as calculated from the State House and the Hyannis station. The greatest difference, which is in the longitude, amounts to 0".16, or about 12 feet; in latitude, the difference is 0".06, = six feet. I have looked over the triangles in that neighbourhood, and feel convinced that the extreme difference should not amount to more than five or six feet. I have therefore come to the conclusion that an error exists in the work of preparation to obtain the bearing and distance from the State House to Highland Light; but as it is an operation of considerable labour to go over the work again, and as the latitude and longitude will be given with sufficient accuracy for every practical purpose, I have thought I should not take the trouble to correct it.

SECTION IV.—OF THE VERTICAL TRIANGLES, AND THE LEVELLING OF THE PRIMITIVE STATIONS OF THE MASSACHUSETTS SURVEY.

I here present a table of the results and comparisons of the measurement of the heights of the primitive trigonometrical stations in the Massachusetts Survey, which Mr. Bou-telle was preparing when he engaged as an assistant to Major Graham in making the North-Eastern Boundary Survey, and which he completed shortly afterwards. Mr. Bou-telle's letter, and the accompanying map of the primitive triangles, will, it is hoped, furnish such a description of the progressive steps of the work, as may be readily comprehended by strangers to the local situation of our primitive triangles, and give a just idea of the operations and calculations.

The levels were gradually united one with another, as the calculations were made from the sea coast, until they reached Fay's Mountain. Having thought a table of the results of the levels as they were carried forward might be of interest, I have copied the one

which follows. The computations are illustrated in Mr. Boutelle's letter. The heights of the first five points were obtained by levelling directly from the middle point between high water and low water mark, in the ordinary manner of levelling for rail-roads and canals.

TABLE OF THE HEIGHTS OF THE PRINCIPAL POINTS AND STATIONS OF MASSACHUSETTS,
ABOVE SEA LEVEL.

Names of Stations.	In what towns situated.	Heights in feet.	Names of Stations.	In what towns situated.	Heights in feet.
Bullock's Neck	On the State line at Seekonk	15.80	Chandler's Hill	Worcester	744.60
Nahant	Lynn	89.90		Mean 748.37	752.14
Coddon's Hill	Marblehead	117.75	Watatic Mt.	Ashburnham	1847.35
Telegraph Hill	Marshfield	205.30	Castle Hill	Saugus	287.99
Hyannis Signal	Barnstable	81.42	Holt's Hill	Andover	425.40
Blue Hill	Milton	635.13		Mean 422.95	420.51
	Mean 635.05	634.97			408.56
Prospect Hill	Waltham	487.52	Prospect Hill	Rowley	264.35
	Mean 482.27	479.67	Ayer's Hill	Haverhill	344.29
		479.63		Mean 339.23	336.69
Prospect Hill	Hingham	243.23			336.71
Scituate Hill	Cohasset	186.13	Powow Hill	Salisbury	325.23
		180.71		Mean 328.02	330.81
	Mean 179.98	177.14	Rail-cut Hill	Gloucester	204.21
		175.93		Mean 204.68	203.63
Monk's Hill	Kingstown	312.86			206.20
	Mean 312.93	313.00	Salisbury Marsh	State line, Salisbury	4.61
Sprague's Hill	Bridgewater	193.54		* Mean 5.86	7.12
	Mean 192.26	189.21	State House, (Top of Cupola.)	Boston	247.97
Red Brush Hill	Wrentham	458.74		Mean 248.84	249.71
	Mean 455.86	452.98	Tuft's Hill	New Braintree	1185.45
Joe's Rock	Wrentham	484.64		Mean 1179.30	1173.47
	Mean 485.84	487.04			1178.96
Beacon Pole Hill	Cumberland R. I.	515.19	Hawe's Hill	Barre	1280.52
	Mean 516.58	517.98		Mean 1285.00	1289.49
Fay's Mountain	Westborough	707.35	Packard's Mountain	New Salem	1282.41
	Mean 706.87	706.39		Mean 1277.82	1273.24
Bald Hill	Douglass	710.57	Colonel's Mountain	Palmer	1166.45
		710.17		Mean 1171.67	1173.95
	Mean 713.56	719.94			1174.60
Allum Pond Station	Douglass	777.31	Mount Grace	Warwick	1631.35
	Mean 777.61	781.24		Mean 1628.21	1625.06
		776.27	Bear Mountain	Wendell	1278.89
Mount Daniel	Webster	785.31		Mean 1281.33	1283.78
	Mean 785.07	784.82	Walnut Hill	Charlemont	1885.87
N. W. cor. of R. I.		633.22		Mean 1887.94	1890.02
Hatchet Hill	Connect't line, Southbridge	1013.38	More's Hill	Goshen	1713.08
	Mean 1015.94	1019.92	High Ridge	Williamsburg	1479.88
		1014.54	French's Hill	Peru	2241.95
Mugget Hill	Charleton	1017.63		Mean 2239.40	2236.85
	Mean 1011.62	1008.92	Saddle Mountain	Adams	3506.16
		1008.30		Mean 3505.50	3504.85
Hasnebumskit Hill	Paxton	1407.42			
Wacushett Mount'n	Princeton	2013.00			
		2022.63			
	Mean 2022.02	2030.44			

* This may be considered as a test station. It stands upon a rock, in the marsh, probably from one to two feet above high water;—the tide ebbs here about ten feet at an average.

Names of Stations.	In what towns situated.	Heights in feet.	Names of Stations.	In what towns situated.	Heights in feet.
Spruce Mountain	Adams Mean 2588.38	2587.83 2588.94	Mount Lincoln	Pelham Mean 1245.92	1246.30 1244.57 1246.89
Jilson's Hill	State line, Rowe Mean 2108.61	2108.28 2108.94	S. end of Base line	Hatfield Mean 169.59	170.89 170.67 167.22
Berlin Mt.	Berlin, N. Y. Taconic Mt's. Mean 2814.35	2813.11 2815.59	N. end of Base line	Deerfield Mean 220.02	219.89 220.13
Clarksburg Mt.	Clarksburg	2272.34	The difference in the height of the termini of the base line determined at the time it was measured, is equal to 49.55 feet.		
Dug Hill	Blandford Mean 1622.48	1621.24 1623.72	Manomet Hill	Plymouth Mean 394.25	392.04 396.46
Becket	Becket Mean 2193.87	2192.10 2195.65	Alden's Hill	Middleboro Mean 177.66	176.02 176.16 180.80
Perry's Peak	Richmond Mean 2089.01	2084.26 2093.47 2089.31	Baur's Hill	Sandwich	297.27
Jackson's Hill	Blandford Mean 1717.21	1716.71 1717.71	Falmouth signal	Falmouth	192.78
Seymour's Mount'n	Sandisfield Mean 1698.05	1699.00 1697.11	Great Hill	Rochester	126.98
Bald Peak	Mount Washington Mean 2623.65	2623.66 2622.49 2624.81	Mendal's Hill	Fairhaven	146.29
Winchell's Mount'n	Granville Mean 1362.45	1360.20 1364.11 1363.03	Copecut Hill	Fall-River Mean 355.00	353.54 356.47
Proven's Mountain	West-Springfield Mean 664.83	666.77 662.90	N. Bedford Court House. Upper part of windows.	New Bedford	164.88
Mount Tom	Northampton Mean 1213.63	1214.01 1215.61 1211.69 1212.34 1214.54	Rhode Island signal	Portsmouth R. I.	286.17
Hilliard's Knob	In line between Amherst and Granby Mean 1119.74	1123.71 1119.21 1118.84 1116.76 1120.17	Pocasset Hill	Tiverton R. I.	323.26
Rattlesnake Mt.	Connecticut line Wilbraham Mean 1076.79	1074.19 1079.40	Fall River signal	Fall River Mean 259.05	258.53 259.58
Peaked Mountain	Monson Mean 1238.77	1243.47 1237.63 1235.21	* Toweset station. Top of signal	Rhode Island line, Swanzey	28.88
Hitchcock's Hill	Wales Mean 1189.64	1190.91 1188.38	College Hill	Providence R. I.	204.22
Mount Esther	Whately Mean 995.12	993.63 996.62	Great Rock Hill	Rehoboth Mean 247.77	249.69 245.86
			Great Meadow Hill	Rehoboth Mean 265.54	265.77 265.31
			King's Rock. Top sig. This signal was	State line, Swanzey probably 25 feet high	103.17
			South end of verification Base }	Seekonk Mean 57.39	60.02 54.77
			German's Hill	Yarmouth	136.48
			Mount Hope	Bristol R. I. Mean 216.41	217.18 215.75

I have thus given the results of our calculations of the heights of the hills, and mountains in Massachusetts, following the straggling manner in which they succeeded each other, that a better judgment might be formed of the degree of confidence to be placed in the accuracy of the work. I would however mention, that when the calculations had been carried to Colonel's Mountain, in Palmer, they were extended from thence, in a northerly and westerly direction, to the extreme western part of the State, and thence, re-

* This may also be considered as a test signal. It stands near tide water, and although the height of the signal has never been exactly measured, yet I have no hesitation in saying, that the height we have found it to be cannot in the extreme differ from truth more than six or eight feet.

turning upon the southerly side of the State, again to Colonel's Mountain. We there compared them, by redetermining the height of the mountain from the returning levels, having traversed, in their zigzag course, a distance of several hundred miles. The result follows:—

Mean height of Colonel's Mountain, obtained from the levels carried directly from tide water,	-	-	-	-	-	-	-	-	-	1171.67	feet.	} 13.13 feet, difference.
From the return levels,	-	-	-	-	-	-	-	-	-	1184.80	feet.	

From the calculation of reciprocal observations, the mean refractions were found to vary from one-tenth to one-twentieth of the arc, upon the earth's surface, contained between the two stations from which the reciprocal observations had been made. It may be worthy of remark, that, in the western portion of the state, the refractions appeared to be much more regular than they were in the eastern,—rarely exceeding one-twelfth, or falling short of one-sixteenth of the contained arc. This phenomenon is probably owing to the trigonometrical stations being much more elevated, above the country which surrounds them, in the western portion of the State, than in the eastern. The western portions of the state may be said to be mountainous.

Probably I ought to mention, before I leave this subject, that the ascertaining of the heights of the trigonometrical stations was deemed of secondary importance; and, as a considerable portion of each day is unsuitable for making accurate observations, our general practice has been to measure the vertical angles at those times,—employing the more suitable states of the atmosphere for measuring the azimuthal angles. It is possible that mistakes may occasionally have been made in registering our field notes,—such as marking an angle as elevated when it should be depressed, and the contrary,—as we did not make repeated measurements of the vertical angles. These mistakes, however, can only take place when the two stations are near the same level. In other conditions, they can be corrected from a knowledge of the apparent or comparative height of the stations, at the time of calculating them.

The following description of the levelling operations has been drawn up, at my request, by Mr. Charles Boutelle, one of the assistants in the Massachusetts survey.

“In order to exhibit the whole matter, in relation to the chain of levels carried across this state in the course of the trigonometrical survey, I present herewith some farther statements and examples, illustrating the system pursued, and the modes of computation employed. I also send a map of the triangles, to show the relative positions of the points observed.

“The zenith distances were observed with the vertical repeating circle of the theodolite with which the azimuth angles were measured. It is graduated to 5'', and has four verniers. It was customary to make from six to eight repetitions, according to circumstances. Owing to a want of confidence in the accuracy of these observations in the early part of the survey, comparatively few observations were taken, and some stations, therefore, depend upon single observations only, for their heights. It will be noticed, however, that where reciprocal and single observations are brought to bear upon the same station, there is generally a close coincidence in the results.

"The formulæ used are found in Malortie's Topography, vol. i., p. 131—135. They are as follows. (I have changed a few of the symbols for greater convenience.)

"1st. To reduce the angles as observed to the tops of the signals. At the point A,

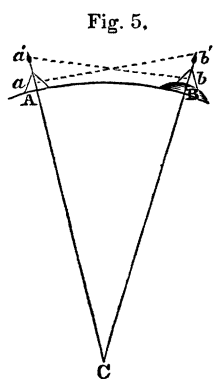


Fig. 5.

(Fig. 5,) an observer with the telescope at a observes the zenith distance* ($= 90^\circ \pm$ the angle of depression or elevation, as the case may be,) of the top of the signal b' at the point B. At B, with the telescope b, he observes the Z. D. of the top of the signal a' at the point A. It is obvious that the angles, measured at the points a and b, are eccentric, and must be reduced to what they would be at a' and b' , before they can be used in computing the difference of level between the points A and B.† To effect this, let the distance aa' , or the difference in height between the top of the signal and the telescope at the station A, be represented by ψ ; the Z. D. measured at a by Δ , and the arc AB in ft, by K , then

$$(1.) \quad d = \frac{\psi \sin \Delta R''}{K}$$

$$(2.) \quad \delta = \Delta + d.$$

And the same at the station B, where $\psi = bb'$; Δ the Z. D. measured at b, and $\delta' = \Delta + d$. d being the correction in seconds to be applied to the observed angle.

"2d. To find the mean refraction in seconds. Let r = refraction; C the angle at the centre of the earth, or, which is the same thing, the arc A B contained between the two stations, expressed in terms of the arc, and let δ and δ' = corrected zenith distances; then

$$(3.) \quad r = \frac{C}{2} - \left(\frac{\delta + \delta' - 180^\circ}{2} \right)$$

"This formula is founded upon the proposition, that if the sum of the two depressions be taken from the measure of the intercepted terrestrial arc, half the remainder is the refraction. If one of the objects is elevated, on account of the smallness of the arc, then if the depression be taken from the sum of the contained arc and elevation, half the remainder will be the refraction. The value in seconds of the arc A B may be found, with sufficient accuracy, by using the mean radius of the earth as given in the books.

"3d. To find the difference of level. Let H represent the difference of level between the tops of the signals at A and B; then

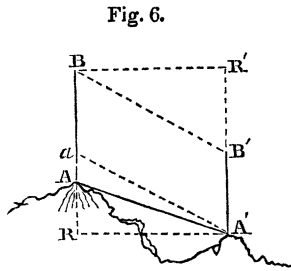
$$(4.) \quad H = K \tan \left(\frac{\delta' - \delta}{2} \right) \quad \text{or} \quad (5.) \quad H = \frac{K \sin \frac{1}{2} (\delta' - \delta)}{\cos \frac{1}{2} (\delta' - \delta + c)}$$

"The first of these two formulæ, although not exact, is yet sufficiently accurate for all practical purposes, and is the one which has been used in our calculations.

* We shall, hereafter, instead of zenith distance, frequently use the initials Z. D.

† The angles may, in like manner, be reduced to the height of the telescope of the instrument at the respective stations, or to the copper bolts which mark the stations, whichever may best suit the notions of the computer.

“Having the difference of level of the tops of the signals, that of the copper bolts marking the station points is found by applying a correction to H equal to the difference in height of the two signals. This correction is + (plus) when the shortest signal is elevated, and — (minus) when it is depressed, as may be seen in the annexed figure, (Fig. 6,) in which $B' R'$ = difference of level between tops of signals, and $A R$ = difference of level between bolts, but $B' R'$ is greater than $A R$ by the quantity $A a$, which is the difference in height of the signals $A B$ and $A' B'$ above the station marks, and which is to be added or deducted, therefore, as indicated above.



“Having thus given the formulæ used where reciprocal observations have been made, I here subjoin an example of their application to practice in this survey.

“At Nahant station, in Lynn, Sept. 14th, 1834, at 4 $\frac{1}{4}$, P. M., the measured zenith distance of the top of the signal on Blue Hill, in Milton, was 89° 45' 35''.84.

Height of the telescope above the copper bolt marking Nahant station, = 4.83 ft.

Height of top of signal above copper bolt at Blue Hill station, . . . = 31.92

$$\psi = 27.09$$

“At Blue Hill station in Milton, Oct. 31st, 1834, at 2 $\frac{1}{3}$, P. M., the measured zenith distance of the top of the signal at Nahant station in Lynn was 90° 25' 44''.58.

Height of telescope above the copper bolt marking Blue Hill station, = 13.63 ft.

Height of top of signal above copper bolt at Blue Hill station, . . . = 34.93

$$\psi = 21.30$$

By (1.) At Blue Hill station, $\Delta = 90^\circ 25' 44''.58$

$$\psi = 21.30 \text{ ft.}$$

$$\psi = 21.30 \quad \log. = 1.3283796$$

$$\Delta = 90^\circ 25' 44''.58 \quad \sin = 9.9999878$$

$$R'' = \quad \log. = 5.3144251$$

$$K = 94402.74 \text{ ar. co. log.} = 5.0250154$$

$$d = 46''.54 = 1.6678079$$

At Nahant station, $\Delta = 89^\circ 45' 35''.84$

$$\psi = 27.09$$

$$\psi = 27.09 \quad \log. = 1.4328090$$

$$\Delta = 89^\circ 45' 35''.84 = 9.9999962$$

$$R'' = 5.3144251$$

$$K \quad \text{ar. co. log.} = 5.0250154$$

$$d = 59''.19 = 1.7722457$$

$$\begin{array}{rcl}
 (2.) \quad \Delta & = & 90^\circ 25' 44''.58 \\
 d & = & \quad + 46.54 \\
 \hline
 \delta & = & 90 \quad 26 \quad 31.12 \\
 \delta' & = & 89 \quad 46 \quad 35.03 \\
 \hline
 \delta - \delta' & = & \quad 39 \quad 56 \quad .09 \\
 \hline
 (4) \quad \frac{\delta - \delta'}{2} & = & 19' 58''.05 \tan = 7.7640366 \\
 K & = & 94402.74 \log. = 4.9749846 \\
 H & = & 548.31 \log. = 2.7390212 \\
 \hline
 \Delta & = & 89^\circ 45' 35''.84 \\
 d & = & \quad + 59.19 \\
 \hline
 \delta' & = & 89 \quad 46 \quad 35.03 \\
 \delta & = & 90 \quad 26 \quad 31.12 \\
 \hline
 \delta + \delta' - 180^\circ & = & \quad 13 \quad 06 \quad .15 \\
 \hline
 (3.) \quad \frac{\delta + \delta' - 180^\circ}{2} & = & 6' 33''.08 \\
 \frac{\dot{C}^*}{2} & = & \quad 7 \quad 46 \quad .11 \\
 \hline
 r = \frac{10}{128} C & = & .078 \quad C = \quad 1' 13''.03
 \end{array}$$

"The difference of level, therefore, between the tops of the signals at Blue Hill and Nahant stations being 548.31 ft., and the signal on Blue Hill being three feet higher above the bolt than that in Nahant, this quantity must be deducted from H , and the remainder, $= 548.31 - 3 = 545.31$ ft., will be the elevation of the bolt at Blue Hill above the bolt at Nahant.

"The bolt at Nahant has been ascertained, by levelling, to be 89.83 ft. above the mean level of the sea; therefore $89.83 + 545.31 = 635.14$ ft. will be the height of Blue Hill station above the sea by this observation.

"Where but one observation has been made, the following formula was applied; namely, $H = K \cot \left(\Delta + r - \frac{C}{2} \right)$; in which H = difference of level between the telescope at the place of observation, and the top of the signal observed. The difference of level of the bolts is obtained by applying a correction to H , equal to the difference between the

* To determine the value of C , we divide K by the value of a second deduced from the mean radius of the earth as determined by Mr. Ivory, its log $= 2.0054651$; thus:

$$\begin{array}{rcl}
 K & = & \log. 4.9749846 \\
 1'' & = & \log. 2.0054651 \\
 \hline
 C & = & 932.22 = 2.9695195
 \end{array}$$

height above the bolts of the signal observed and the telescope at the place of observation. This correction is + (plus) when the signal observed is depressed, and — (minus) when it is elevated.

“The refraction, r , was obtained, in the easterly part of the state, by using .0784 C as a mean coefficient, derived from about twenty reciprocal observations. In the interior, the mean refraction was found to be less than on the sea-coast, and a mean coefficient of .06978 C was employed.

“The following example will illustrate the foregoing remarks:—

“At Blue Hill station, Oct. 30th, 1834, the telescope standing 13.63 ft. above the copper bolt, the measured zenith distance of Coddon’s Hill station, in Marblehead, was $90^{\circ} 22' 11''.25$. Height of signal at Coddon’s above the bolt = 29.83 feet.

“By the formula given above,

$$\begin{array}{rcl}
 \Delta & = & 90^{\circ} 22' 11''.25 \\
 r = .0784 C & = & + \quad 1 \quad 41 \quad .62 \\
 & & \hline
 & & 90 \quad 23 \quad 52 \quad .87 \\
 \frac{C}{2} & = & - \quad 10 \quad 48 \quad .09 \\
 & & \hline
 \Delta + r - \frac{C}{2} & = & 90 \quad 13 \quad 04 \quad .78 \cot = 7.5803348 \\
 K & = & 131259.69 \quad \log. = 5.1181314 \\
 & & \hline
 H & = & 499.42 \quad \log. = 2.6984662 \\
 \text{Correction for reduction to bolts} & = & 29.83 - 13.63 = + \quad 16.20 \\
 & & \hline
 & & 515.62 \text{ ft.}
 \end{array}$$

“Blue Hill is, therefore, elevated 515.62 ft. above Coddon’s. The height of Coddon’s Hill station, above the mean level of the sea, has been ascertained, by levelling, to be 117.75 feet; therefore, $117.75 + 515.62 = 633.37$ feet is the result obtained, by this observation, for the height of Blue Hill station above the mean level of the sea. This is 1.77 feet less than that obtained by the reciprocal observation from Nahant, and giving the single observation one third weight, the mean will be 634.55 feet for the height of the station.

“In this manner the levels have been carried over the state, and the results have been highly satisfactory, proving sufficiently accurate to warrant the assertion that no point is in error more than six feet, and most of them are much nearer. A fair specimen of the work may be seen in the following table, in which the levels, taken from the sea at four stations distant from each other, are brought to bear upon one station. The four stations are Nahant, Telegraph Hill, Hyannis and Bullock’s Neck stations; (the accompanying map of the triangles will show the relative positions of the stations named in the following table;) and the point where the levels from them meet is at Copecut station, in Fall River. In this table, where the sign + is prefixed to the numbers in the column of differences of level, it signifies that the station last named on the same line is elevated, and the contrary sign is used to denote a depression.

No.	STATIONS.	Difference of Level.	Refr. C = 1.	Mean Height above Sea.	
<i>Nahant Station, above Sea, 89.83 feet.</i>					
1.	Nahant and Blue Hill,	+ 545.31	.078	635.05	
2.	Nahant and Prospect Hill, Hingham, . .	+ 153.42	.077	243.25	
3.	Prospect Hill, Hingham and Monk's Hill,	+ 69.77	.087		
<i>Telegraph Hill, above Sea 205.30 feet.</i>					
4.	Telegraph and Monk's Hills,	+ 107.56	.105	312.94	Mean height of Monk's Hill
5.	Prospect Hill, Hingham and Sprague's Hill,	— 49.72	.077	192.24	from (3) and (4) = 312.94.
6.	Blue and Sprague's Hills,	— 445.90	.071		
7.	Monk's and Sprague's Hills,	— 118.90	.050	394.20	Mean height of Sprague's
8.	Telegraph and Manomet Hills,	+ 186.64	.086		from (5), (6), (7) = 192.24.
9.	Monk's and Manomet Hills,	+ 83.53	.100	177.64	Mean height of Manomet
10.	Manomet and Alden's Hills,	— 218.23	.081		from (8), (9) = 394.20.
11.	Monk's and Alden's Hills,	— 136.77	.083	177.64	Mean height of Alden's from
12.	Sprague's and Alden's Hills,	— 11.46	.064		(10), (11), (12) = 177.64.
<i>Hyannis Station, above Sea 81.42 feet.</i>					
13.	Hyannis and Falmouth stations,	+ 111.36	.091	192.78	
14.	Falmouth station and Great Hill,	— 65.80	.079	126.98	
15.	Great and Mendal's Hills,	+ 19.31	.094	146.29	
<i>Bullock's Neck Station, above Sea 15.80 feet.</i>					
16.	Bullock's Neck and Rhode Island stations, .	+ 270.37	.058	286.17	
17.	Rhode Island and Mount Hope stations, .	— 70.42	.058	215.75	
18.	Mount Hope and Fall River stations, . .	+ 41.87	.094	257.62	
19.	Fall River and Copecut stations,	+ 96.47	}	354.69	Height above sea of Copecut
20.	Mendall's Hill and Copecut stations, . .	+ 210.18			from (19) = 354.09.
21.	Alden's Hill and Copecut stations, . . .	+ 175.88			Do. from (20) = 356.47.
					Do. from (21) = 353.52.

"It will be seen that the height of Alden's Hill is determined from Nahant and Telegraph stations, the height of Mendal's Hill from Hyannis station, and the height of Fall River station from Bullock's Neck; and that the extremes of difference of these levels, in meeting on Copecut station, are only three feet. The levels from Nahant and Telegraph stations, meeting at Monk's Hill, differ but $\frac{1.8}{100}$ of a foot. This is the closest and fairest test which can be applied to trigonometrical levelling, and the result in this case is as near as could be reasonably expected.

"I will close this letter by a table, in which we start at Nahant station, and proceed, by *one chain* of levels, to Saddle Mount, in Berkshire, the highest point in the state.

No.	STATIONS.	Difference of Level.	Refr. C = 1.	Height above Sea.	Mean Height above Sea.
1.	Nahant,			89.83	89.83
1.	Nahant and Blue Hill,	+ 545.31	.078	635.14	635.05
2.	Blue Hill and Fay's Mount,	+ 72.22	.079	707.36	707.36
3.	Fay's Mount and Bumsket Hill,	+ 700.07	.100	1407.43	1407.43
4.	Bumsket and Tuft's Hills,	— 233.95	.063	1173.48	1179.30
5.	Tuft's and Hawes' Hills,	+ 110.19		1283.67	1285.00
6.	Hawes' Hill and Packard's Mount,	— 2.59	.059	1281.08	1277.82
7.	Packard's and Bear Mounts,	+ 1.07	.049	1282.15	1281.33
8.	Bear Mount and Walnut Hill,	+ 604.54	.065	1886.69	1887.94
9.	Walnut Hill and Saddle Mount,	+ 1616.91	.091	3503.60	3505.50

"It will be seen by this table, that had the observed height of Saddle Mount depended on this chain of levels alone, it would have been about two feet less than the mean; and the greatest difference between any height as given by this table, and the same height as determined from a mean of all the observations brought to bear upon it, is at Tuft's Hill station, where the difference is nearly six feet."

SECTION V.—LATITUDES AND LONGITUDES OF TWENTY-SEVEN PLACES IN MASSACHUSETTS, ETC., AS DETERMINED, SOLELY, BY ASTRONOMICAL OBSERVATIONS, WITH PREFATORY REMARKS, BY R. T. PAINE.

THE operations for ascertaining the latitudes and longitudes by astronomical observation were accomplished, without assistance, by Robert Treat Paine, Esq., under an appointment from Governor Lincoln. Mr. Paine's account of his labours and their results is contained in the following report, made to Governor Everett, of which a copy was furnished to me, and is now, for the first time, presented for publication.

TO HIS EXCELLENCY EDWARD EVERETT, ESQ.

SIR,—I have the honour herewith to present to the Executive of Massachusetts a table of the latitudes and longitudes of twenty-seven places, (twenty-three of which are in the said state, and the remainder in its immediate vicinity,) deduced by me from many thousand observations, made in pursuance of a resolve of the Legislature thereof, passed in 1830.

Being, sir, of opinion that a concise and popular explanation of the manner in which these observations were made would not be altogether unacceptable either to the Executive or the Legislature, I have taken the liberty to preface the report with the following remarks.

With the highest respect,

I am, your Excellency's obedient servant,

R. T. PAINE.

Boston, March, 1838.

FOR the determination of the position of any place on the surface of the earth, it is necessary that two things should be known; namely, its latitude or distance north or south from the imaginary line or circle called the equator, and its longitude or distance east or west from another imaginary line or circle, which passes at right angles to the former, through some other place, which is assumed to be the first meridian.

Several methods have been proposed by which the latitude might be ascertained, but no one is as simple, or more accurate than that of measuring, with a suitable instrument, the altitude of the sun or other celestial body whose declination is known, when on or near the meridian. Indeed, so simple is this method, that when only an approximation to the truth is wanted, as at sea, a single altitude will give a result sufficiently exact; but

when precision is required, many altitudes of the sun, or, what is preferable, of stars, passing both to the north and south of the observer's zenith, must be taken, and the corrections of the altitudes for refraction and parallax, and of the stars' declinations, for aberration, nutation, and precession, must be carefully computed. The *accurate* solution of the problem, therefore, even by this method, is no longer easy, but requires much labour and time. Indeed, there is reason to doubt whether *perfect* accuracy can be obtained. In the Royal Observatory at Greenwich, which was established in 1675, are some of the largest astronomical instruments ever constructed, yet, within a few years, its latitude has been diminished one second; and the late director of that establishment has remarked, "that an error of half a second is even now not at all improbable, though it is rather unlikely that it should exceed that quantity." In the latitude of Greenwich, one second is equivalent to one hundred and one feet and five inches, and in Massachusetts to one hundred and one feet and three inches; the length of a degree of latitude, increasing gradually, but constantly, as we recede from the equator, in consequence of the ellipticity or flattening of the earth.

For the determination of the difference of the longitudes of two places there are many methods. But several of them, such as the eclipses of the moon, and of the satellites of Jupiter, and the distance of the moon from the sun or stars, are too uncertain to be resorted to when accuracy is requisite; and, indeed, no one of the various methods that have been proposed is altogether without its disadvantages. A single observation on the interval between the transit of the moon and a star may not give a result more accurate than a lunar distance, though the mean of several hundred of such observations, made in the course of several years, would probably be very correct. Eclipses of the sun, occultations of stars, and transits of the two inferior planets, are of very rare occurrence; moreover, the longitude of the place of observations will be affected by the errors in the tables of the sun, moon, planet, or star, unless corresponding observations were made on the same eclipse, occultation, or transit, at some other place, whose longitude is known, whereby the amount of such error may be discovered and taken into consideration. The explosion of gunpowder or of rockets, or the sudden extinction of a bright light on some conspicuous eminence, (a method which was used with much success in the recent determination of the longitudes of Greenwich and Paris,) requires the services of several practical astronomers; and the last method, which, in theory, is equally simple and perfect, that of transporting the time of one place to another by a chronometer, is liable to the objection, that although this is a beautiful, and even wonderful instrument, it is still not sufficiently perfect to be implicitly relied on. Such, however, is the comparative beauty and simplicity of this method, that it will hereafter be frequently adopted, especially for the determination of the difference of the longitudes of places not remote from each other; whilst, in order to avoid the difficulty above mentioned, several of these instruments must be employed, and the comparisons between the places must be repeated until, from the mean of all the observations, a result shall be obtained, nearly as accurate as can be hoped for.

It is, therefore, obvious that the determination of the exact longitude is not less difficult than of latitude. Indeed, it has been pronounced one of the most so of all the problems in practical astronomy. The astronomical observatories at Greenwich and Paris are not only two of the oldest, but perhaps the two most important in Europe: it

would seem, therefore, a matter of course that the difference of their longitudes should, long since, have been ascertained; yet, principally on account of the difficulty above mentioned, so late as the year 1775, a century after the establishment of the former, the supposed difference was erroneous by five and a half seconds of time; as it was then considered $9^m 16^s$; in 1780, Gen. Roy deduced from the trigonometrical survey of England, made under his direction, $9^m 18^s.8$; thirty-five years ago $9^m 20^s$ was used; and within a few years from observations on about one thousand transits of the moon and a star, and many on the explosion of rockets, $9^m 21^s.5$ was deduced, which is the quantity now used.

As the place from which longitude shall be reckoned is altogether arbitrary, it follows, as might have been expected, that different nations have fixed upon different places for the first meridian. Thus the English universally, and the Americans generally, reckon from the observatory at Greenwich, the French from that at Paris, the Germans from Berlin or Ferro, the Spanish from Cadiz, &c. It is much to be desired that some one place should be generally agreed on, and nothing but national prejudice has hitherto prevented the attainment of this object. It is highly probable that Greenwich will eventually be considered the first meridian, as already the two greatest commercial nations of the earth reckon from it, and as the observatory there has long been by common consent pronounced the most important and distinguished. Indeed, an eminent and gifted astronomer of France has remarked of the observations made at Greenwich, that if by a great revolution the observations made every where else were lost, and those only should be saved, there would be found in them materials sufficient to rear anew almost the entire edifice of modern astronomy.

I am well aware that in a few American maps the capitol at Washington has been adopted as the first meridian, for no other reason than that it is the building in which the national congress assembles. This step appears to me to be highly injudicious. The number of first meridians is not only thus increased, but the position of that building is not by any means well ascertained; indeed, from the observations made in its vicinity, on the annular eclipses of the sun of 1791, 1811 and 1831, I have deduced for its longitude a quantity *greater by more than six miles* than that usually assigned to it.

Moreover, if we consider Greenwich as the first meridian, the longitude of every part of the continent of America will be west; but if Washington or any other city in the United States be fixed on, part of our territory will be in east and part in west: this would be troublesome and inconvenient, and might be productive of serious error.

For these reasons the longitude of the following places has been reckoned from Greenwich, which I sincerely hope, will, in the construction of the map, be adopted as the first meridian.

For the determination of the latitudes of the places in the following table, the method above mentioned was generally adopted. Altitudes of the pole star (α Ursæ Minoris) were, however, measured when in any part of its circuit around the pole, and very many altitudes of southern stars were obtained, when at a short distance from the meridian to which they were reduced by the rules of spherical trigonometry. The altitudes of every star but the polar were reduced separately, and also of that when it was less than three hours from its upper or lower culmination.

It will be seen that the observations for the determination of the latitudes of all the

places but three, have been quite numerous; and of these three, it is believed a sufficient number was made for denoting their situations; they are not situate within the limits of the state, but so near that it is probable they will be comprehended by the map.

For the determination of the longitude of the State House in Boston, use was made of the observations there or in its vicinity, on every recent eclipse of the sun or occultation of a star, on which a corresponding observation was made, in some place in Europe. But, as usual, they were very few. It is believed that the only occultations seen there and in Europe, within several years past, were those of α Tauri in 1829 and 1830, and the only eclipse of the sun that of May 15th, 1836. From these three corresponding observations, I was able to deduce the errors of the lunar tables, and thus obtain for the longitude of the State House a quantity, less by only a tenth of a second, than that deduced by Doctor Bowditch in 1812, from six other observations, made between the years 1743 and 1807, which longitude he considered better ascertained than that of any other place in the United States.

The determination of the differences of the longitudes of the other twenty-six places, and of the State House, by observations, or the explosion of gunpowder or of rockets, on the summits of some of our highest hills, was reluctantly abandoned. As before observed, this method is not only very simple and accurate, but probably requires less time than any other; but it could not be adopted without the assistance of at least two persons well versed in practical astronomy, and it will be recollected that I have not had, at any time, the services of even one assistant.

The method, the last of those above enumerated, was therefore employed; namely, that of transporting the time of one place to another by chronometers; and, with a few exceptions, all the results stated in the following tables were in this manner obtained. In order to diminish as much as possible the probability of error, from any injury that might be sustained by the timekeepers whilst undergoing transportation, or from the local time at any place of observation not having been precisely ascertained, several of these instruments were generally used, (and even they were rarely depended on for a longer interval than twenty-four hours, and never for more than two days,) and the comparisons between the places in question were repeated again and again, until a result was obtained apparently deserving of confidence. Thus, on referring to the following table, it will be seen that the longitude of the First Congregational Church in Northampton was ascertained by seventy-four chronometers and twenty-four comparisons with the State House, or the Antiquarian Hall, in Worcester; that the longitude of the new Court House in Barnstable was ascertained by fifty-nine chronometers and seventeen comparisons, &c., &c. And it will be, moreover, noticed, that the difference between the greatest and least longitudes of any place thus determined was seldom equal to two seconds of time.

The local time at any place was ascertained, not by a transit instrument, but by an artificial horizon of quicksilver, and a reflecting instrument that had been constructed for me by the celebrated Troughton. It was usual to measure at least six altitudes of a bright star in the east, and directly as many of another star in the west, or the reverse. As it was but seldom that the errors of the chronometers deduced from the two sets of altitudes differed two-thirds of a second, and as the mean of these errors was used, there is every reason to suppose that the local time was thus well ascertained.

	Latitudes, North.
Hartford, Conn., State House,	41° 45' 59".1
Holmes Hole, Windmill west of the village,	27 15.3
Lowell, St. Ann's Church,	42 38 47.6
Monomoy Point, Light House,	41 33 30.8
Nantucket, South Tower,	16 56.0
New Bedford, Mariners' Church,	38 6.3
Newburyport, Harris Street Church,	42 48 32.1
Northampton, First Congregational Church,	19 8.0
Pittsfield, " " "	26 55.0
Plymouth, Court House,	41 57 28.5
Portsmouth, N. H., Unitarian (stone) Church,	43 4 34.6
Providence, R. I., University Hall,	41 49 31.9
Salem, East India Marine Hall,	42 31 18.9
Sandwich, First Congregational Church,	41 45 31.0
Springfield, Court House,	42 6 1.2
Taunton, Trinitarian Congregational Church,	41 54 8.3
Truro, Cape Cod Lighthouse,	42 2 22.2
Williamstown, Congregational Church near College,	42 50.6
Worcester, Antiquarian Hall,	16 12.6

Longitudes of twenty-seven places in Massachusetts, &c.

	FROM THE STATE HOUSE.		FROM GREENWICH.	
	In Degrees.	In Time.	In Degrees.	In Time.
Boston, State House,			71° 4' 9".0	4h 44m 16s.60
Amherst, College Chapel,	+ 1° 27' 26".9	+ 5m 49s.79	72 31 35.9	50 6.39
Barnstable, Court House,	— 0 45 33.0	— 3 2.20	70 18 36.0	41 14.40
Bristol, R. I., Episcopalian Church,	+ 0 13 10.5	+ 0 52.69	71 17 19.5	45 9.29
Cambridge, First Congregational Church,	+ 0 3 29.1	+ 0 13.94	71 7 38.1	44 30.54
Dedham, " " "	+ 0 6 40.2	+ 0 26.68	71 10 49.2	44 43.28
Gloucester, First Independent Church,	— 0 23 50.0	— 1 35.33	70 40 19.1	42 41.27
Greenfield, Second Congregational Church,	+ 1 32 23.0	+ 6 9.53	72 36 32.0	50 26.13
Hartford, Conn., State House,	+ 1 36 36.3	+ 6 26.42	72 40 45.3	50 43.02
Holmes Hole, Windmill,	— 0 27 31.2	— 1 50.08	70 36 37.8	42 26.52
Lowell, St. Ann's Church,	+ 0 14 48.3	+ 0 59.22	71 18 57.3	45 15.82
Monomoy Point, Light House,	— 1 4 3.6	— 4 16.24	70 0 5.4	40 0.36
Nantucket, South Tower,	— 0 57 56.9	— 3 51.79	70 6 12.2	40 24.81
New Bedford, Mariners' Church,	— 0 8 19.7	— 0 33.31	70 55 49.4	43 43.29
Newburyport, Harris Street Church,	— 0 11 21.9	— 0 45.46	70 52 47.1	43 31.14
Northampton, First Congregational Church,	+ 1 34 12.0	+ 6 16.80	72 38 21.0	50 33.40
Pittsfield, " " "	+ 2 11 56.1	+ 8 47.74	73 16 5.1	53 4.34
Plymouth, Court House,	— 0 23 41.4	— 1 34.76	70 40 27.6	42 41.84
Portsmouth, N. H., Stone Church,	— 0 18 19.2	— 1 13.28	70 45 49.8	43 3.32
Providence, R. I., University Hall,	+ 0 20 39.0	+ 1 22.60	71 24 48.0	45 39.20
Salem, East India Marine Hall,	— 0 10 12.3	— 0 40.82	70 53 56.7	43 35.78
Sandwich, First Congregational Church,	— 0 33 42.0	— 2 14.80	70 30 27.0	42 1.80
Springfield, Court House,	+ 1 31 38.3	+ 6 6.55	72 35 47.3	50 23.15
Taunton, Trinitarian Congregational Church,	+ 0 1 55.5	+ 0 7.70	71 6 4.5	44 24.30
Truro, Cape Cod Lighthouse,	— 1 0 0.3	— 4 0.02	70 4 8.7	40 16.58
Williamstown Congregational Church,	+ 2 9 10.5	+ 8 36.70	73 13 19.5	52 53.30
Worcester, Antiquarian Hall,	+ 0 44 1.2	+ 2 56.08	71 48 10.2	47 12.68
West Longitudes are designated thus +				
East " " " " —				

Boston, (State House.) For the Latitude.

1828 and 1829,	194	altitudes of \odot	gave	42° 21' 21".7
1833, April,	12	"	"	"	24.7
"	11	"	"	"	23.9
"	14	"	"	"	19.9
"	10	"	"	"	24.6
September,	12	"	"	"	23.2
1834, March,	8	"	"	"	26.5
May,	20	"	α Ursæ Minoris	gave	24.8
"	34	"	"	"	"	.	.	.	24.5
1835, "	17	"	α η	"	"	.	.	.	23.4
"	6	"	"	"	"	.	.	.	26.4
June,	15	"	"	"	"	.	.	.	24.9
May,	18	"	α Ursæ Minoris	"	"	.	.	.	26.4
June,	10	"	"	"	"	.	.	.	25.3
1837, October,	8	"	β Ceti	"	"	.	.	.	21.9
"	14	"	α Ursæ Minoris	"	"	.	.	.	20.5
"	2	"	"	"	"	.	.	.	21.7
"	7	"	β Ceti	"	"	.	.	.	24.5
November,	10	"	"	"	"	.	.	.	22.4
"	16	"	α Ursæ Minoris	"	"	.	.	.	20.8
December,	2	"	β Ceti	"	"	.	.	.	20.2
"	12	"	α Ursæ Minoris	"	"	.	.	.	20.9
"	10	"	\odot	"	"	.	.	.	20.3
"	12	"	β Ceti	"	"	.	.	.	21.1
"	2	"	α Ursæ Minoris	"	"	.	.	.	24.6
"	15	"	β Orionis	"	"	.	.	.	24.8
"	20	"	α Ursæ Minoris	"	"	.	.	.	25.5
"	12	"	β Ceti	"	"	.	.	.	19.1
"	16	"	\odot	"	"	.	.	.	18.9
"	16	"	α Ursæ Minoris	"	"	.	.	.	19.5
"	13	"	"	"	"	.	.	.	24.6
"	12	"	β Orionis	"	"	.	.	.	20.3
"	5	"	β Ceti	"	"	.	.	.	23.8
"	4	"	α Ursæ Minoris	"	"	.	.	.	24.0
"	12	"	"	"	"	.	.	.	23.2
"	13	"	β Orionis	"	"	.	.	.	26.2
"	10	"	δ	"	"	.	.	.	24.8
"	12	"	α Ursæ Minoris	"	"	.	.	.	21.1
431 altitudes of \odot and Southern Stars,									22.24
205 " " Northern "									23.36
Mean of 636 observations,									42° 21' 22".7

Boston, (State House.) For the Longitude.

Several years since, the longitude of the State House was deduced by Dr. Bowditch from observations made in its vicinity, on the following phenomena; namely,—

The Transit of ξ , Nov. 5th, 1743,	gave	.	.	.	4 ^h 44 ^m 19 ^s .2
Eclipse " \odot , Aug. 5th, 1766,	"	.	.	.	17.5
Transit " φ , June 3d, 1769,	"	.	.	.	18.0
Eclipse " \odot , June 24th, 1778,	"	.	.	.	14.9
" " " April 3d, 1791,	"	.	.	.	18.0
" " " June 16th, 1806,	"	.	.	.	11.8
Mean Longitude from these six observations,					4 ^h 44 ^m 16 ^s .6

And it was the opinion of Dr. Bowditch, that “this longitude is more accurately ascertained than that of any other place in the United States.”

From the observations on the occultations of α Tauri, a star of the first magnitude, on August 21st, 1829, and January 5th, 1830, made in many of the observatories of Europe, and in this city by myself, I deduced the longitude of the State House to be $4^h 44^m 14^s.6$. And from the observations made at Greenwich and at Dorchester, by a friend, and at Providence by myself, on the great eclipse of the sun, of May 15th, 1836, I deduced $4^h 44^m 19^s.6$.

The mean of these three observations is $16^s.3$, or only three-tenths of a second less than the longitude determined by Dr. Bowditch; and the mean of the nine observations is only one-tenth of a second less, a quantity too small to be noticed.

I therefore state the longitude of the State House (west from the Royal Observatory at Greenwich) to be $4^h 44^m 16^s.6$, or, in degrees, $71^\circ 4' 9''.0$.

<i>Amherst, (College Chapel.) For the Latitude.</i>											
1832, October,	26	altitudes of the ☉ gave	-	-	-	-	-	-	42° 22' 11".4		
1834, August,	14	“ α Aquilæ	-	-	-	-	-	-	13 .6		
“	14	“ α Ursæ Minoris	-	-	-	-	-	-	13 .8		
“	4	“ “	-	-	-	-	-	-	13 .2		
“	12	“ β Ceti	-	-	-	-	-	-	12 .5		
“	9	“ α Ursæ Minoris	-	-	-	-	-	-	12 .0		
1836, “	10	“ “	-	-	-	-	-	-	11 .4		
“	10	“ α Aquilæ	-	-	-	-	-	-	11 .9		
“	10	“ α Ursæ Minoris	-	-	-	-	-	-	14 .8		
	62	“ ☉ and Southern Stars	-	-	-	-	-	-	12 .19		
	47	“ “ Northern “	-	-	-	-	-	-	13 .11		
	Mean of 109 observations,								42 22 12 .6		
<i>Amherst, (College Chapel.) For the Longitude.</i>											
1832, October,	By 2 chronometers east of Mansion House, Northampton,							-	-	0 ^m 27 ^s .76	
1833, October,	2	“	“	“	“	“	-	-	-	28 .42	
“	2	“	“	“	“	“	-	-	-	28 .27	
1834, August,	2	“	“	“	“	“	-	-	-	27 .11	
“	2	“	“	“	“	“	-	-	-	26 .88	
1835, August,	4	“	“	“	“	“	-	-	-	27 .69	
September,	4	“	“	“	“	“	-	-	-	28 .59	
1836, August,	3	“	“	“	“	“	-	-	-	28 .36	
	Mean of 21 chronometers, east of Mansion House, Northampton,							-	-	—	27 .96
	Longitude of Northampton,							-	-	+ 6	17 .72
	Longitude of Amherst College Chapel,							-	-	+ 5	49 .76

II.

1832, October,	By 2 chronometers west of State House,								+ $5^m 50^s.03$
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III.

1832, October,	By 2 chronometers west of Antiquarian Hall, Worcester,								+ $2^m 53^s.82$
	Longitude of Ant. Hall, Worcester,								+ 2 56 .08
	Longitude of Amherst College Chapel,								+ 5 49 .90
	By 2 chronometers compared directly with State House,								+ $5^m 50^s.03$
2	“	“	“	“	“	“	“	“	49 .90
21	“	“	“	“	“	“	“	“	49 .76
	Mean of 25 chronometers,								+ 5 49 .79
	Or from Greenwich,								4 50 6 .39

SURVEY OF MASSACHUSETTS.

Barnstable, (New Court House.) For the Latitude.

1832, April,	39	altitudes of ☉ gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	41° 42'	7".8
"	21	" " "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.9
Sept.	18	" " "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.9
1835, June,	6	" 2 α ♂ gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.0
"	8	" β ♂ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.3
"	8	" α Ursæ Minoris gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		1.8
"	6	" β ♀	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		4.3
"	11	" 2 α ♂	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		6.4
"	4	" β ♂	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.3
"	12	" α ♀	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.9
"	15	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.8
"	15	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.6
July,	20	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.0
1837, June,	10	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		10.4
"	6	" α ♀	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.1
"	6	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		6.9
"	10	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.6
"	12	" α ♂	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.8
"	10	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.2
"	10	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		10.3
"	10	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.8
"	10	" α ♀	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.1
159 altitudes of ☉ and Southern Stars,															-	-	-	-	7.09
108 " " Northern "															-	-	-	-	7.53
Mean of 267 observations,															-	-	-	-	41° 42' 7".3

Barnstable, (New Court House.) For the Longitude.

1832, September,	By 2	chronometers	east of State House,	-	-	-	-	-	3 ^m 3 ^s .44
“	2	“	“	“	“	-	-	-	0.36
1835, June,	4	“	“	“	“	-	-	-	2.70
July,	4	“	“	“	“	-	-	-	2.40
1837, June,	4	“	“	“	“	-	-	-	2.98
“	4	“	“	“	“	-	-	-	1.99
July,	4	“	“	“	“	-	-	-	2.95
“	4	“	“	“	“	-	-	-	1.35
“	3	“	“	“	“	-	-	-	1.88
Mean of 31 chronometers,					-	-	-	-	3 ^m 2 ^s .28

II.

1832, September,	By 2 chronometers east of Plymouth Court House,	-	-	-	-	1 ^m 28 ^s .03
“	2 “ “ “ “ “	-	-	-	-	26.81
1835, June,	4 “ “ “ “ “	-	-	-	-	28.13
“	4 “ “ “ “ “	-	-	-	-	27.22
1837, June,	4 “ “ “ “ “	-	-	-	-	28.44
“	4 “ “ “ “ “	-	-	-	-	26.73
July,	4 “ “ “ “ “	-	-	-	-	26.72
	Mean of 24 chronometers,	-	-	-	-	— 1 27.44
	Longitude of Plymouth,	-	-	-	-	— 1 34.76
	Longitude of Barnstable,	-	-	-	-	— 3 2.20

III.

1832, April,	By 4 chronometers east of the Mariners' Church, in New Bedford,	— 2 ^m 28 ^s .44
	Longitude of New Bedford, - - - - -	— 33.31
	Longitude of Barnstable, - - - - -	— 3 1.75
	By 31 chronometers compared with the State House, - - - - -	3 ^m 2 ^s .28
	24 " " " Plymouth, - - - - -	2.20
	4 " " " New Bedford, - - - - -	1.75
	Mean of 59 chronometers, - - - - -	— 3 2.20
	Or, from Greenwich, - - - - -	4 41 14.40

Bristol, R. I., (Episcopal Church.) For the Latitude.

1832, March,	44 altitudes of ☉ gave - - - - -	41° 40' 1". 0
1837, August,	2 " β Ceti gave - - - - -	3 . 6
"	8 " α Ursæ Minoris gave - - - - -	4 . 9
Sept.	12 " " " " - - - - -	3 . 0
"	12 " α Aquilæ " - - - - -	5 . 9
	58 altitudes of ☉ and Southern Stars, - - - - -	2.10
	20 " " Northern " - - - - -	3.76
	Mean of 78 observations, - - - - -	41° 40' 2". 9

Bristol, R. I., (Episcopal Church.) For the Longitude.

1832, April,	By 4 chronometers east of Providence University Hall, - - - - -	29 ^s .65
1837, August,	4 " " " " " - - - - -	30.77
	Mean of 8 chronometers, - - - - -	— 30.21
	Longitude of Providence, - - - - -	+ 1 22.60
	Longitude of Bristol, - - - - -	+ 0 52.39

II.

1832, April,	By 4 chronometers west of New Bedford, - - - - -	1 ^m 26 ^s .59
1837, Sept.	4 " " " " " - - - - -	26.01
	Mean of 8 chronometers, - - - - -	+ 1 26.30
	Longitude of New Bedford, - - - - -	— 33.31
	Longitude of Bristol, - - - - -	+ 52.99
	By 8 chronometers compared with Providence University Hall - - - - -	+ 0 ^m 52 ^s .39
	8 " " " New Bedford Mariners' Church, - - - - -	0 52.99
	Mean of 16 chronometers, - - - - -	+ 0 52.69
	Or, from Greenwich, - - - - -	4 45 9.29

Cambridge, the First Congregational Church, (Mr. Newell's.) For the Latitude.

1835, May,	11 altitudes of α Ursæ Minoris gave - - - - -	42° 22' 19".7
"	21 " α η " " - - - - -	19.8
"	16 " α Ursæ Minoris " - - - - -	22.1
"	17 " " " " - - - - -	19.9
"	10 " " " " - - - - -	18.4
"	22 " α η " " - - - - -	21.7
"	20 " α Ursæ Minoris " - - - - -	21.4
"	12 " α η " " - - - - -	21.3
"	10 " α Ursæ Minoris " - - - - -	18.9

Gloucester, (First Independent or Universalist Church.) For the Latitude.

1834, Sept.	12	altitudes of α Ursæ Minoris	gave	-	-	-	-	-	-	42° 36' 48".4
"	13	"	☉	"	-	-	-	-	-	49 .1
"	4	"	α Ursæ Minoris	"	-	-	-	-	-	46 .9
1835, July,	20	"	"	"	-	-	-	-	-	43 .0
"	10	"	α Aquilæ	"	-	-	-	-	-	42 .0
"	8	"	η Ophiuchi	"	-	-	-	-	-	44 .2
"	20	"	α Ursæ Minoris	"	-	-	-	-	-	45 .1
"	12	"	"	"	-	-	-	-	-	41 .3
"	20	"	"	"	-	-	-	-	-	41 .9
"	12	"	α Aquilæ	"	-	-	-	-	-	42 .0
Oct.	4	"	β Ceti	"	-	-	-	-	-	43 .0
"	12	"	α Ursæ Minoris	"	-	-	-	-	-	45 .4
"	10	"	"	"	-	-	-	-	-	46 .6
"	12	"	β Orionis	"	-	-	-	-	-	42 .5
59 altitudes of ☉ and Southern Stars,										44 .03
110 " " Northern "										44 .32
Mean of 169 observations,										42° 36' 44".2

Gloucester, (First Independent Church, formerly Dr. Murray's.) For the Longitude.

1834, Sept.	By 4 chronometers east of the State House,	-	-	-	-	-	1 ^m 35 ^s .68
1835, July,	4 " " " " " "	-	-	-	-	-	36 .68
Aug.	4 " " " " " "	-	-	-	-	-	35 .46
Oct.	4 " " " " " "	-	-	-	-	-	34 .54
"	4 " " " " " "	-	-	-	-	-	34 .80
1836, July,	4 " " " " " "	-	-	-	-	-	35 .93
Aug.	4 " " " " " "	-	-	-	-	-	34 .90
Mean of 28 chronometers,							1 ^m 35 ^s .43

II.

1834, Sept.	By 2 chronometers east of Salem Lafayette Hotel,						-	-	-	54 ^s .32
1835, Aug.	4	"	"	"	"	"	-	-	-	54.27
Oct.	4	"	"	"	"	"	-	-	-	54.67
"	4	"	"	"	"	"	-	-	-	54.11
1836, July,	4	"	"	"	"	"	-	-	-	55.06
"	4	"	"	"	"	"	-	-	-	54.89
Mean of 22 chronometers,										— 54.57
Or, adding the longitude of Salem Lafayette Hotel,										— 40.59
										— 1 ^m 35.16

III.

1834, Sept.	By 2 chronometers east of Newburyport Harris Street Church,	-	-	49 ^s .91
1835, Oct.	4 " " " " " "	-	-	49 .55
1836, July,	4 " " " " " "	-	-	50 .34
Mean of 10 chronometers,				49 .94
Or, adding the longitude of Newburyport,				45 .46
				1 ^m 35 ^s .40
By 28 chronometers compared with the State House,				1 ^m 35 ^s .43
22	" " " Salem,	-	-	35 .16
10	" " " Newburyport,	-	-	35 .40
Mean by 60 chronometers from State House,				1 ^m 35 ^s .33
"	" " " Greenwich,	-	-	4 ^h 42 41 .27

II.

1833, April,	By 2 chronometers west of Newburyport, Harris Street Church,	-	1 ^m 45 ^s .19
1834, June,	2 " " " " " " -	-	45 .32
"	2 " " " " " " -	-	45 .70
	Mean of 6 chronometers,	- - - - -	+ 1 ^m 45 ^s .41
	Longitude of Newburyport,	- - - - -	— 45 .46
	Longitude of Lowell,	- - - - -	+ 59 .95
	By 28 chronometers compared with the State House,	- - - - -	+ 59 .52
	6 " " " Newburyport,	- - - - -	59 .95
	Mean of 34 chronometers,	- - - - -	+ 59 .60
	Reduction to the church,	- - - - -	— 0 .38
	Longitude of the church by 34 chronometers,	- - - - -	+ 0 ^m 59 ^s .22
	Or, from Greenwich,	- - - - -	4 45 15 .82

Monomoy Point, or Cape Malabar, (Lighthouse.) For the Latitude.

1831, February,	28 altitudes of ☉ gave	- - - - -	41° 33' 30".1
"	24 " " " -	- - - - -	29 .8
"	10 " α Canis Majoris gave	- - - - -	31 .6
1835, June,	20 " α Ursæ Minoris " -	- - - - -	27 .5
"	7 " β η " -	- - - - -	33 .5
"	6 " α η " -	- - - - -	28 .2
"	10 " α Ursæ Minoris " -	- - - - -	34 .2
"	10 " " " " -	- - - - -	28 .7
"	4 " α η " -	- - - - -	34 .7
"	10 " β π " -	- - - - -	33 .3
"	6 " β η " -	- - - - -	35 .5
"	10 " α η " -	- - - - -	30 .4
"	11 " α Ursæ Minoris " -	- - - - -	32 .2
	105 altitudes of ☉ and Southern Stars,	- - - - -	31".11
	51 " " Northern " -	- - - - -	30 .06
	Mean of 156 observations,	- - - - -	41° 33' 30".8

Monomoy Point, (Lighthouse.) For the Longitude.

1831, February,	By 2 chronometers east of State House,	- - - - -	4 ^m 16 ^s .51
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II.

1835, June,	By 4 chronometers east of Barnstable Court House,	- - - - -	1 ^m 15 ^s .17
"	4 " " " " " -	- - - - -	13 .51
	Mean of 8 chronometers,	- - - - -	— 1 14 .34
	Longitude of Barnstable,	- - - - -	— 3 2 .20
	Longitude of Monomoy,	- - - - -	— 4 16 .54

III.

The whole of the annular eclipse of the sun which happened February 12th, 1831, was observed by me at this Lighthouse; but, in consequence of clouds, only the beginning was visible in Dorchester. By combining the observations made there with those made at the Lighthouse on the beginning and end, and the formation and rupture of the ring, I deduced for the longitude of the Lighthouse — 4^m 15^s.5.

Longitude by 28 chronometers compared with State House,	-	-	-	— 45°.49
" " " Salem,	-	-	-	45.41
Mean of 46 chronometers,	-	-	-	— 45.46
Or, from Greenwich,	-	-	-	4 43 31.14

Northampton, (First Congregational Church.) For the Latitude.

The Mansion House, the place where the following observations were made, being 3''.39 south of the church, that quantity must be constantly added.

1832, Oct.	15	altitudes of ☉ gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	42° 19'	2".7						
1833, June,	6	" α η gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.9						
Oct.	7	" β Ceti gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.3						
"	8	" ☉ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		0.0						
"	6	" α Canis Majoris gave	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		0.2						
"	10	" ☉ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		3.5						
Dec.	14	" "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.3						
"	6	" "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		11.4						
"	20	" "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		6.3						
"	11	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.8						
"	33	" " "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.5						
"	19	" ☉ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.4						
"	4	" β Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.0						
"	21	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.6						
"	20	" ☉ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		9.6						
"	15	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		5.2						
"	21	" ☉ "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.6						
1834, July,	12	" α Aquilæ	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		4.7						
"	12	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		3.6						
Aug.	5	" β Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.0						
"	12	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		6.5						
"	8	" β Ceti	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		1.3						
"	10	" α Ursæ Minoris	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		3.7						
"	12	" " "	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.9						
1836, Sept.	10	" " "	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		7.2						
"	10	" α Aquilæ	"	-	-	-	-	-	-	-	-	-	-	-	-	-	-		8.6						
																		<hr/>							
																		182 altitudes of ☉ and Southern Stars,	4.72						
																		145 " " Northern, "	4.52						
																		<hr/>							
																		327	Latitude of Mansion House,	-	-	-	-	42° 19'	4".62
																			Add reduction to church,	-	-	-	-		3.39
																		<hr/>							
																		Mean of 327 observations, latitude of church, -	-	-	-	-	42° 19'	8".0	

Northampton, (First Congregational Church.) For the Longitude.

The Mansion House being $0^{\circ}.92$ west of the church, that quantity must be constantly added, to reduce the observations to the meridian of the church.

1832, Oct.	By 2	chronometers	west	of	the	State	House,	-	-	-	6 ^m 17 ^s .80
1833, June,	2	"	"	"	"	"	"	-	-	-	19.05
July,	2	"	"	"	"	"	"	-	-	-	17.87
Dec.	3	"	"	"	"	"	"	-	-	-	17.68
1834, July,	2	"	"	"	"	"	"	-	-	-	18.82

1834, July,	By 3 chronometers west of the State House,						-	-	-	-	6 ^m 18 ^s .97
August,	4	"	"	"	"	"	-	-	-	-	18.20
1835, "	4	"	"	"	"	"	-	-	-	-	16.78
Sept.	4	"	"	"	"	"	-	-	-	-	17.96
1836, August,	4	"	"	"	"	"	-	-	-	-	17.04
Sept.	4	"	"	"	"	"	-	-	-	-	17.01
1837, "	4	"	"	"	"	"	-	-	-	-	18.85
"	4	"	"	"	"	"	-	-	-	-	16.94
Mean of 42 chronometers,											+ 6 17.81

II.

1832, Oct.	By 2 chronometers west of Antiquarian Hall, Worcester,						-	-			3 ^m 21 ^s .58
1833, June,	2	"	"	"	"	"	-	-	-	-	22.82
July,	2	"	"	"	"	"	-	-	-	-	21.51
Oct.	2	"	"	"	"	"	-	-	-	-	21.72
1834, July,	2	"	"	"	"	"	-	-	-	-	22.61
1835, Aug.	4	"	"	"	"	"	-	-	-	-	20.66
Sept.	4	"	"	"	"	"	-	-	-	-	21.91
1836, Aug.	3	"	"	"	"	"	-	-	-	-	21.29
Sept.	3	"	"	"	"	"	-	-	-	-	21.14
1837, Oct.	4	"	"	"	"	"	-	-	-	-	20.98
"	4	"	"	"	"	"	-	-	-	-	22.72
Mean of 32 chronometers,											+ 3 21.54
Add Longitude of Worcester,											+ 2 56.08
											+ 6 ^m 17 ^s .62
By 42 chronometers compared with the State House,											+ 6 17.81
32 " " " Worcester,											17.62
Mean of 74 chronometers, (Mansion House,)											+ 6 17.72
Add reduction to the church,											- 0 0.92
Longitude of the church by 74 chronometers,											+ 6 16.80
Or, from Greenwich,											4 50 33.40

On the 22d of August, 1836, the immersion of the star ϵ τ behind the dark limb of the moon was observed, by a friend, in Dorchester, and by myself at the Mansion House. The immersion took place at Dorchester, mean time, at 10^h 23^m 20^s.90, and at the Mansion House at 10^h 14^m 57^s.46; from which I have deduced the difference of longitude between the two places of observation to be 6^m 17^s.20. To which add 0^s.69 for the longitude of Dorchester, and subtract the reduction 0^s.92 \pm 0^s.23, and we have, for the difference of the longitude of the State House and church, 6^m 16^s.97, which differs only one-sixth of a second from the results given by the seventy-four chronometers.

Pittsfield, (First Congregational Church.) For the Latitude.

1832, October,	16	altitudes of \odot gave	-	-	-	-	-	-	-	42° 26' 51".4
1833, June,	6	" α η gave	-	-	-	-	-	-	-	57.9
"	10	" " "	-	-	-	-	-	-	-	58.5
Oct.	8	" α Canis Majoris gave	-	-	-	-	-	-	-	48.7
"	9	" \odot "	-	-	-	-	-	-	-	53.4
Nov.	20	" " "	-	-	-	-	-	-	-	56.3
"	12	" α Ursæ Minoris "	-	-	-	-	-	-	-	53.8

II.

1836, July,	By 3 chronometers east of Newburyport Church,	-	-	-	-	-	-	28 ^s .19
"	3 " " " " " " " "	-	-	-	-	-	-	27.76
1837, Aug.	4 " " " " " " " "	-	-	-	-	-	-	27.79
	Mean of 10 chronometers,	-	-	-	-	-	-	— 27.90
	Longitude of Newburyport,	-	-	-	-	-	-	— 45.46
								— 1 13.36
	By 10 chronometers compared with the State House,	-	-	-	-	-	-	— 1 ^m 13 ^s .19
	10 " " " " Newburyport,	-	-	-	-	-	-	13.36
	Mean of 20 chronometers,	-	-	-	-	-	-	— 1 13.28
	Or, from Greenwich,	-	-	-	-	-	-	4 43 3.32

Providence, (University Hall, or Old College.) For the Latitude.

1832, Jan.	42 altitudes of ☉ gave	-	-	-	-	-	-	41° 49' 27".4
March,	37 " " " " " " " "	-	-	-	-	-	-	32.7
"	32 " " " " " " " "	-	-	-	-	-	-	32.4
"	46 " " " " " " " "	-	-	-	-	-	-	36.1
Nov.	26 " " " " " " " "	-	-	-	-	-	-	27.7
1836, May,	11 " α Ursæ Minoris gave	-	-	-	-	-	-	37.3
"	12 " α ηξ " " " "	-	-	-	-	-	-	32.2
"	10 " α Ursæ Minoris " " " "	-	-	-	-	-	-	34.5
"	17 " " " " " " " "	-	-	-	-	-	-	35.2
"	13 " α ηξ " " " "	-	-	-	-	-	-	28.6
"	14 " " " " " " " "	-	-	-	-	-	-	27.2
"	14 " α Ursæ Minoris " " " "	-	-	-	-	-	-	29.4
Oct.	10 " " " " " " " "	-	-	-	-	-	-	30.4
1837, Aug.	12 " " " " " " " "	-	-	-	-	-	-	32.0
"	12 " α Aquilæ " " " "	-	-	-	-	-	-	32.6
	234 altitudes of ☉ and Southern Stars,	-	-	-	-	-	-	31.24
	74 " " Northern " " " "	-	-	-	-	-	-	33.15
	Mean of 308 observations,	-	-	-	-	-	-	41° 49' 31".9

Providence, (University Hall.) For the Longitude.

1832, March,	By 2 chronometers west of the State House,	-	-	-	-	-	-	+ 1 ^m 22 ^s .44
Nov.	2 " " " " " " " "	-	-	-	-	-	-	22.50
1836, May,	4 " " " " " " " "	-	-	-	-	-	-	22.56
"	4 " " " " " " " "	-	-	-	-	-	-	22.09
"	4 " " " " " " " "	-	-	-	-	-	-	22.57
Oct.	4 " " " " " " " "	-	-	-	-	-	-	22.08
"	4 " " " " " " " "	-	-	-	-	-	-	22.32
1837, Aug.	4 " " " " " " " "	-	-	-	-	-	-	23.14
"	4 " " " " " " " "	-	-	-	-	-	-	23.31
	Mean of 32 chronometers,	-	-	-	-	-	-	+ 1 22 ^s .57

II.

1832, April,	By 4 chronometers west of New Bedford Mariners' Church,	-	-	-	-	-	-	+ 1 ^m 56 ^s .24
1837, Sept.	4 " " " " " " " "	-	-	-	-	-	-	56.77
	Mean of 8 chronometers,	-	-	-	-	-	-	+ 1 56.50
	Add Longitude of New Bedford,	-	-	-	-	-	-	— 33.31
								+ 1 23.19

Mean of 27 chronometers compared with Northampton,	-	-	+ 6 ^m	6 ^s .66
12 " " " State House,	-	-		6.38
12 " " " Worcester,	-	-		6.49
Mean of 51 chronometers,	-	-	+ 6	6.55
Or, from Greenwich,	-	-	4 50	23.15

Taunton, (Trinitarian Congregational Church.) For the Latitude.

1833, March,	12	altitudes of ☉ gave	-	-	-	-	-	-	41° 54' 9".0
"	12	" " " "	-	-	-	-	-	-	9.8
Aug.	12	" " " "	-	-	-	-	-	-	8.0
1834, Oct.	12	α Ursæ Minoris gave	-	-	-	-	-	-	5.7
"	10	β Ceti " "	-	-	-	-	-	-	8.0
"	11	α Ursæ Minoris " "	-	-	-	-	-	-	5.3
"	9	☉ " "	-	-	-	-	-	-	8.6
1835, Sept.	18	α Ursæ Minoris " "	-	-	-	-	-	-	10.2
"	6	β Ceti " "	-	-	-	-	-	-	9.8
"	10	α Ursæ Minoris " "	-	-	-	-	-	-	9.4
"	8	β Ceti " "	-	-	-	-	-	-	9.3
"	10	α Ursæ Minoris " "	-	-	-	-	-	-	9.4
"	7	β Ceti " "	-	-	-	-	-	-	8.5
"	10	α Ursæ Minoris " "	-	-	-	-	-	-	6.7
"	14	β Orionis " "	-	-	-	-	-	-	7.9
"	4	α Ursæ Minoris " "	-	-	-	-	-	-	5.5
1837, Sept.	8	β Ceti " "	-	-	-	-	-	-	9.9
"	8	α Ursæ Minoris " "	-	-	-	-	-	-	8.0
98 altitudes of ☉ and Southern Stars,									8".79
83 " " Northern " "									7.85
Mean of 181 observations,									41° 54' 8".3

Taunton, (Trinitarian Congregational Church, nearly opposite the County House.) For the Longitude.

1833, March,	By 2 chronometers west of State House,	-	-	-	-	-	7 ^s .73
Aug.	2 " " " " "	-	-	-	-	-	7.73
1834, Oct.	2 " " " " "	-	-	-	-	-	8.15
1835, Sept.	4 " " " " "	-	-	-	-	-	7.44
"	4 " " " " "	-	-	-	-	-	8.34
1837, Sept.	4 " " " " "	-	-	-	-	-	7.69
Mean of 18 chronometers,							7.84

II.

1833, March,	By 2 chronometers west of New Bedford Mariners' Church,	-	41 ^s .20
Aug.	2 " " " " "	-	40.83
1834, Oct.	2 " " " " "	-	41.37
1835, Sept.	4 " " " " "	-	40.77
"	4 " " " " "	-	40.26
1837, "	4 " " " " "	-	41.15
Mean of 18 chronometers,			+ 40.87
Longitude of New Bedford,			- 33.31
			+ 7 ^s .56

By 18 chronometers compared with the State House,	-	-	-	+	7 ^s .84
18 " " " New Bedford,	-	-	-	+	7.56
Mean of 36 chronometers,	-	-	-	+	7 ^s .70
Or, from Greenwich,	-	-	-	4 44	24.30

Truro, (Cape Cod or Highland Lighthouse.) For the Latitude.

1832, April,	8 altitudes of ☉ gave	-	-	-	-	-	-	42° 2' 26" .4
1835, Aug.	1 " α Ursæ Minoris gave	-	-	-	-	-	-	21.5
"	3 " " " " " " " "	-	-	-	-	-	-	21.6
"	5 " α Aquilæ " " " " " "	-	-	-	-	-	-	20.2
"	25 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	20.0
"	18 " α Aquilæ " " " " " "	-	-	-	-	-	-	19.6
"	21 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	20.8
"	20 " α Aquilæ " " " " " "	-	-	-	-	-	-	20.8
"	10 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	25.4
"	12 " β Ceti " " " " " "	-	-	-	-	-	-	24.4
"	22 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	24.8
"	10 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	25.7
"	13 " α Aquilæ " " " " " "	-	-	-	-	-	-	23.0
"	18 " β Ceti " " " " " "	-	-	-	-	-	-	20.2
"	16 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	23.4
"	12 " β Ceti " " " " " "	-	-	-	-	-	-	22.0
"	14 " α Ursæ Minoris " " " " " "	-	-	-	-	-	-	22.1
	106 altitudes of ☉ and Southern Stars,	-	-	-	-	-	-	21.70
	122 " " Northern " " " " " "	-	-	-	-	-	-	22.65
	Mean of 228 observations,	-	-	-	-	-	-	42° 2' 22" .2

Truro, (Cape Cod Lighthouse.) For the Longitude.

1835, Aug.	By 4 chronometers east of the State House,	-	-	-	-	-	-	— 3 ^m 59 ^s .19
"	4 " " " " " " " "	-	-	-	-	-	-	60.89
"	3 " " " " " " " "	-	-	-	-	-	-	58.95
"	3 " " " " " " " "	-	-	-	-	-	-	59.75
	Mean of 14 chronometers,	-	-	-	-	-	-	— 3 59.74

II.

1832, April,	By 2 chronometers east of Barnstable Court House,	-	-	-	-	-	-	— 57 ^s .33
1835, July,	4 " " " " " " " "	-	-	-	-	-	-	58.46
"	4 " " " " " " " "	-	-	-	-	-	-	58.37
	Mean of 10 chronometers,	-	-	-	-	-	-	— 58.20
	Longitude of Barnstable,	-	-	-	-	-	-	— 3 2.20
		-	-	-	-	-	-	— 3 60.40
	By 14 chronometers compared with the State House,	-	-	-	-	-	-	— 3 59.74
	10 " " " Barnstable,	-	-	-	-	-	-	60.40
	Mean of 24 chronometers,	-	-	-	-	-	-	— 4 0.02
	Or, from Greenwich,	-	-	-	-	-	-	4 40 16.58

SECTION VI.—COMPARISON OF THE RESULTS OF MR. BORDEN'S TRIANGULATION, WITH THOSE OF MR. PAINE'S ASTRONOMICAL OBSERVATIONS.

Having given, in the preceding sections, a detailed statement of the methods used in obtaining the single results of the triangulation, and of the astronomical observations of Mr. Paine, I now proceed to furnish a comparison of the data obtained by the respective processes, in order to show at once the degree of precision of each, and their bearing on the standard values for the reduction of the triangulation, and on the question of the dimensions of the spheroid. The first step is to ascertain the value of the meridional degree, for some cardinal point of middle latitude. For this point I have selected the State House at Boston. I subjoin in a tabular form Mr. Paine's astronomical differences of latitude in arc, and the geodetic differences of latitude in feet, at the standard temperature, referred to the level of the sea.

No.	Names of Stations.	Mr. Paine's Latitude.	Middle Latitude.	Deg. of Meridian in English ft.
	Station of Comparison, Harris St. Church, Newburyport,	42° 48' 32".1		
1	Nantucket South Tower, - - - - -	41 16 56 .0	42° 2' 44".05	364313.17
2	Holmes' Hole Windmill, - - - - -	41 27 15 .3	42 7 53 .70	389.25
3	New Bedford Mariners' Church, - - - - -	41 38 6 .3	42 13 19 .20	348.25
4	Barnstable Court House, - - - - -	41 42 7 .3	42 15 19 .70	420.25
5	Sandwich Church, - - - - -	41 45 31 .0	42 17 1 .55	357.10
	Station of Comparison, Salem East India Marine Hall,	42 31 18 .9		
6	Nantucket South Tower, - - - - -	41 16 56 .0	41 54 7 .45	364253.76
7	Holmes' Hole Wind Mill, - - - - -	41 27 15 .3	41 59 17 .10	340.59
8	New Bedford Church, - - - - -	41 38 6 .3	42 4 42 .60	279.42
9	Sandwich Church, - - - - -	41 45 31 .0	42 8 24 .95	274.76
10	Barnstable Court House, - - - - -	41 42 7 .3	42 6 43 .10	447.68
	Station of Comparison, Highland Light, Cape Cod, -	42 2 22 .2		
11	Nantucket South Tower, - - - - -	41 16 56 .0	41 39 39 .10	364385.00
	Station of Comparison, St. Ann's Church, Lowell, -	42 38 47 .6		
12	New Bedford Mariners' Church, - - - - -	41 38 6 .30	42 8 26 .95	364236.76
	Station of Comparison, Gloucester Church, - - -	42 36 44 .20		
13	Barnstable Court House, - - - - -	41 42 7 .30	42 9 25 .75	364925.68
14	Nantucket South Tower, - - - - -	41 16 56 .00	41 56 50 .10	594.00
15	New Bedford Mariners' Church, - - - - -	41 38 6 .30	42 7 25 .25	738.33
	Station of Comparison, Harris St. Church, Newburyport,	42 48 32 .10		
16	Plymouth Court House, - - - - -	41 57 28 .50	42 23 0 .30	364604.50

Rejecting the last four results in consequence of their difference from the mean, the others give the value of a degree of the meridian in English feet, at the several middle latitudes, as follows:

FIRST RESULT.			SECOND RESULT.		
No.	Middle Latitude.	Length of Degree.	No.	Middle Latitude.	Length of Degree.
1)	42° 2' 44" .05	364313.17 feet.	3)	42° 13' 19" .20	364348.25 feet.
2)	42 7 53 .70	389.25	4)	42 15 19 .70	420.25
6)	41 54 7 .45	253.76	5)	42 17 1 .55	357.10
7)	41 59 17 .10	340.59			
8)	42 4 42 .60	279.42	3)	45 40 .45	3) 1125.60
12)	42 8 26 .95	236.76			
9)	42 8 24 .95	274.76	42	15 13 .48	364375.20
10)	42 6 43 .10	447.68			
8)	336 32 19 .90	8) 2535.39			
	42 4 2 .48	364317.00			

THIRD RESULT.		
11)	41 39 39 10	364385.00

In the absence of the necessary data, to reduce the values thus obtained to the same middle latitude, I referred for the occasion to the table in Rees's Cyclopædia, under the article Degree, which purports to give the value of meridional degrees of the terrestrial spheroid for every degree of latitude, supposing an ellipticity of $\frac{1}{334}$ th of the equatorial radius; and which indicates fifty-seven feet as the increase in the value of the consecutive degrees, from the fortieth to the forty-third degree of latitude. Applying this increase, by arithmetical progression, to each of the foregoing values of the meridional degree, I inferred from them a value corresponding with the middle latitude forty-two degrees; and with the values thus reduced, and giving to each value a weight proportionate to the number of comparisons on which it was based, I obtained 364334 feet for the length of a degree whose middle latitude is 42°. The length of a degree perpendicular to the meridian, at the latitude of the Boston State House, was found from the convergency of the meridians to be 365511.33 feet, which value also was adopted. Its accuracy was tested by applying the results of trigonometrical measurement to the differences of longitude ascertained by Mr. Paine's chronometrical observations. The following values of degrees perpendicular to the meridian were attained in the same manner.

	Feet.
1) From Boston State House and Northampton Church,	365177.60
2) " " " " and Plymouth Court House,	365553.00
3) " " " " and Amherst College Chapel,	365025.00
4) " Springfield Court House and Plymouth Court House,	365888.43
5) " " " " and Highland Light, Cape Cod,	365984.43
6) " Greenfield Church and Gloucester Church,	365420.76
7) " Boston State House and Pittsfield Church,	364193.11
8) " Plymouth Court House and " "	364796.00
9) " Boston State House and Williamstown Church,	364519.42
Sum of the first 6 results,	6) 3149.22
Mean of the first 6 results,	365525.00

The last three of these results are rejected on account of their discrepancy from the mean. The astronomical observations for determining the longitude at Pittsfield and Williamstown are supposed to have been affected by mountain attraction; and I have adopted the value of the degree perpendicular to the meridian, as derived from the incli-

nation of the meridians, in preference to the mean result given by this last table, in consequence of the discrepancies among its single results.

With the values of the meridional and perpendicular degrees thus found, I calculated the equatorial radius, polar semi-axis, and ellipticity of the terrestrial spheroid, and the differences in value of the meridional degrees of $41^{\circ} 21' 30''$, $42^{\circ} 21' 30''$, and $43^{\circ} 21' 30''$. The results are as follows:—

Meridional degree for the latitude of the State House, 364356 feet.

Perpendicular degree for the same latitude, 365511 feet.

Equatorial radius, 20914728 feet = 3961.123 miles.

Polar semi-axis, 20854128 feet = 3949.646 miles.

Ellipticity, $\frac{1}{345}$ nearly of the equatorial radius.*

Length of meridional degrees, the latitude of whose middle point corresponds to

	Feet.	Difference.
$41^{\circ} 21' 30'' =$	364300.96	
		+ 55.04 feet.
$42 \quad 21 \quad 30 =$	364356.00	
		+ 55.22 “
$43 \quad 21 \quad 30 =$	364411.22	

From the calculated differences in the value of these meridional degrees, it was apparent that the increase of 57 feet to the degree, which I had adopted when preparing to deduce the value of the meridional degree from a combination of my proximate results, was not strictly accurate. But as the effect of the resulting error would scarcely have been appreciable, I did not deem it necessary to recalculate the work.

With the data already mentioned, I proceeded to determine the latitude of a *Cardinal* point, namely, the State House at Boston, by comparison with several of the principal stations, as follows:—

No.	Place compared with Boston State House.	Resulting Latitude of Boston State House.
1)	New Bedford Mariners' Church,	$42^{\circ} 21' 29''.84$
2)	Harris Street Church in Newburyport,	29 .95
3)	Salem East India Marine Hall,	30 .38
4)	Saint Ann's Church in Lowell,	30 .82
5)	Barnstable Court House,	31 .23
6)	Sandwich Church in Sandwich,	29 .91
7)	Highland Lighthouse, Cape Cod,	28 .84
8)	Nantucket South Tower,	29 .38
9)	Holmes' Hole Windmill,	30 .43
		<hr/>
		9) 270 .78
		<hr/>
Mean latitude of the State House,		$42^{\circ} 21' 30''.08$
Mr. Paine's Astronomical Result,		42 21 23 .03
		<hr/>
Discrepancy,		7''.05
		<hr/>

The results arrived at from these data, on the final reduction of the triangles of the survey, using for the latitude of the State House $42^{\circ} 21' 30''$, and for its longitude, that which is given by Mr. Paine, $4^{\text{h}} 44^{\text{m}} 16^{\text{s}}.6$ west of Greenwich, are compared with the

* Combining the meridional degree measured in Peru, in latitude $1^{\circ} 30'$, with the meridional degree measured in Massachusetts, they give an ellipticity of $\frac{1}{313}$ th nearly of the equatorial radius.

results of the chronometrical survey of Mr. Paine, in the following tables; which exhibit, also, the number of altitudes of the sun and north and south stars taken by Mr. Paine in determining the latitude of each station, and the number of journeys made by him with chronometers, and of the chronometers used by him on such journeys, in ascertaining its longitude.

Latitudes.

No.	Place in which Station is situated.	Name of Station.	Altitudes observed.	Paine's Latitudes.	Paine, north of Borden.
1	Boston,	State House,	442	42° 21' 22".70	— 7".00
2	Amherst,	College Chapel,	109	42 22 12 .60	— 3 .01
3	Barnstable,	New Court House,	267	41 42 7 .30	+ 1 .23
4	Cambridge,	1st Congregational Church,	201	42 22 21 .30	— 7 .81
5	Dedham,	" " "	198	42 14 52 .30	— 5 .00
6	Greenfield,	2d " " "	169	42 35 16 .30	+ 1 .50
7	Gloucester,	1st Independent Church,	113	42 36 44 .20	— 3 .97
8	Holmes' Hole,	Windmill west of Village,	174	41 27 15 .30	+ 0 .43
9	Lowell,	St. Ann's Church,	300	42 38 47 .60	+ 0 .82
10	Monomoy Point,	Light House,	156	41 33 30 .80	— 4 .20
11	Nantucket,	South Tower Ch.,	260	41 16 56 .00	— 0 .62
12	New Bedford,	Mariners' Church,	322	41 38 6 .30	— 0 .16
13	Newburyport,	Harris Street Church,	202	42 48 32 .10	— 0 .05
14	Northampton,	1st Congregational Church,	327	42 19 8 .00	— 1 .09
15	Pittsfield,	" " "	210	42 26 55 .00	— 0 .59
16	Plymouth,	Court House,	169	41 57 28 .50	+ 1 .94
17	Providence, R. I.,	University Hall,	308	41 49 31 .90	— 2 .58
18	Salem,	East India Marine Hall,	154	42 31 18 .90	+ 0 .38
19	Sandwich,	1st Congregational New Unitarian Church,	139	41 45 31 .00	— 0 .09
20	Springfield,	Court House,	168	42 6 1 .20	— 2 .41
21	Taunton,	Trinitarian Church,	181	41 54 8 .30	— 2 .98
22	Truro,	Cape Cod Lights,	228	42 2 22 .20	— 1 .16
23	Williamstown,	Congregational Church near College,	110	42 42 50 .60	+ 1 .53
24	Worcester,	Antiquarian Hall,	351	42 16 12 .60	— 4 .44
25	Squam,	Light,	38	42 39 46 .08	+ 2 .53
26	Cape Ann,	N. Light, Thatcher's Island,	39	42 38 18 .00	— 3 .78
27	Eastern Point,	Light,	36	42 34 48 .00	— 1 .61
28	Baker's Island,	Light,	64	42 32 11 .40	— 0 .62
29	Hartford, Conn.,	State House,	23	41 45 59 .10	+ 0 .96
30	Cambridge,*	Harvard Observatory Transit,	N. of Boston State House, 52".26		
31	Dorchester,*	Bond's Transit. Ins.	S. of " " " 2' 13 .41		
32	Southwick,*	Holcomb's House,	S. of Springfield C. H., 5 13 .91		

Longitudes.

No.	Place in which Station is situated.	Journeys with Chronometers.	No. of Chronometers used.	Paine's Longitudes.	Paine, west of Borden.
1	Boston,			71° 4' 9".00	+ 0".00
2	Amherst,	9	25	72 31 35 .85	+ 7 .36
3	Barnstable,	18	59	70 18 36 .00	+ 2 .19
4	Cambridge,	6	20	71 7 38 .10	+ 9 .25
5	Dedham,	7	23	71 10 49 .20	— 10 .08
6	Greenfield,	6	14	72 36 31 .95	+ 4 .75
7	Gloucester,	16	60	70 40 19 .05	+ 1 .88
8	Holmes' Hole,	10	36	70 36 37 .80	+ 0 .22
9	Lowell,	14	34	71 18 57 .30	— 4 .74
10	Monomoy Point,	2	10	70 0 5 .40	+ 9 .16

* NOTE BY THE COMMITTEE.—Nos. 30, 31, and 32 are according to Mr. Borden's survey, not having been principal stations of Mr. Paine. Their connexion with the survey is important, from their being the site of a series of independent astronomical observations by Mr. Bond and Mr. Holcomb, which, when reduced, will further test the precision of the two methods employed by Messrs. Paine and Borden.

No.	Place in which Station is situated.	Journeys with Chronometers.	No. of Chronometers used.	Paine's Longitudes.	Paine, west of Borden.
11	Nantucket,	10	28	70° 6' 12".15	— 1 .73
12	New Bedford,	18	54	70 55 49 .35	+ 4 .96
13	Newburyport,	16	46	70 52 47 .10	+ 5 .80
14	Northampton,	24	74	72 38 21 .00	+ 6 .06
15	Pittsfield,	13	39	73 16 5 .10	+ 28 .98
16	Plymouth,	10	33	70 40 27 .60	+ 8 .33
17	Providence, R. I.,	11	40	71 24 48 .00	+ 13 .59
18	Salem,	14	42	70 53 56 .70	+ 3 .67
19	Sandwich,	13	38	70 30 27 .00	+ 13 .81
20	Springfield,	16	51	72 35 47 .25	+ 2 .31
21	Taunton,	12	36	71 6 4 .50	+ 9 .47
22	Truro,	7	24	70 4 8 .70	+ 13 .37
23	Williamstown,	10	28	73 13 19 .50	+ 19 .24
24	Worcester,	15	45	71 48 10 .20	— 2 .84
25	Squam,	2	14	70 41 8 .00	— 4 .33
26	Cape Ann,	2	14	70 34 44 .00	— 4 .05
27	Eastern Point,	By bearing from Gloucester Point.		70 40 12 .75	+ 1 .90
28	Baker's Island,	2	14	70 47 37 .00	+ 8 .59
30	Cambridge,*	West of State House, Boston, . . . 3 6 .42			
31	Dorchester,*	West of " " " . . . 11 .24			
32	Southwick,*	West of Springfield Court House, . . 12 59 .86			

Comparison of the latest elements of the spheroid obtained by Bessel, from the mean of the ten surveys hitherto executed, (see Astr. Nachr., No. 438,) with those derived by me from the triangulation of Massachusetts, combined with Mr. Paine's differences of latitude and longitude.

Elements.	Values from Bessel's Elements.	Borden's values from the Map Survey.	Discrepancy.
Equatorial radius in English feet, - - - -	20923597.14	20914728.00	
" " " miles, - - - -	3962.803	3961.123	1.68 miles.
Polar semi-axis in English feet, - - - -	20853657.16	20854128.00	
" " " miles, - - - -	3949.556	3949.646	0.09 miles.
Degree of the meridian for lat. 42° 21' 30" in feet,	364403.28	364356.00	47.28 feet.
Degree of the perpendicular to the meridian do. do.	365740.21	365511.33	228.88 feet.
Degree of the parallel for lat. 42° 21' 30" do. do.	270262.09	270092.12	169.97 feet.
Latitude of the southernmost point Nantucket, - -	41° 16' 57".19	41° 16' 56".62	0".57
Latitude of the northernmost point Newburyport -	42° 48' 32".00	42° 48' 32".15	0".15

The comparison of Mr. Paine's differences of longitude by chronometers with those of the triangulation, reduced respectively by Bessel's latest elements of the spheroid, and those obtained from the Massachusetts survey, gives the following results for the several stations, referred to the Boston State House.

No.	Station.	Paine West of Borden in time.	Borden West of Bessel in time.	No.	Station.	Paine West of Borden in time.	Borden West of Bessel in time.
1	Boston,	0.00	0.00	15	Pittsfield,	+ 1.93	+ 0.33
2	Amherst,	+ 0.49	+ 0.22	16	Plymouth,	+ 0.56	— 0.06
3	Barnstable,	+ 0.15	— 0.12	17	Providence,	+ 0.91	+ 0.05
4	Cambridge,	+ 0.62	+ 0.01	18	Salem,	+ 0.24	— 0.02
5	Dedham,	— 0.67	+ 0.02	19	Sandwich,	+ 0.92	— 0.08
6	Greenfield,	+ 0.31	+ 0.23	20	Springfield,	+ 0.15	+ 0.23
7	Gloucester,	+ 0.13	— 0.05	21	Taunton,	+ 0.63	+ 0.00
8	Holmes' Hole,	+ 0.01	— 0.06	22	Truro—Highland Light,	+ 0.89	— 0.15
9	Lowell,	— 0.32	+ 0.03	23	Williamstown,	+ 1.28	+ 0.32
10	Monomoy Light,	+ 0.61	— 0.16	24	Worcester,	— 0.19	+ 0.11
11	Nantucket,	— 0.11	— 0.14	25	Squam Light,	— 0.29	— 0.05
12	New Bedford,	+ 0.33	— 0.02	26	Thacher's Island Light,	— 0.27	— 0.07
13	Newburyport,	+ 0.39	— 0.03	27	Eastern Point Light,	+ 0.13	— 0.05
14	Northampton,	+ 0.40	+ 0.24	28	Baker's Island Light,	+ 0.57	— 0.04

* See note on preceding page.

In consequence of a remark of Mr. Hassler, that differences of longitude obtained by transportation of chronometers do not possess the requisite precision for determining the elements for the reduction of the triangulation, and are consequently unfit for geodetic purposes, I was induced to compare the results derived from all the chronometric differences of longitude, which, according to the plan pursued, would produce a weight worth noticing, (except Williamstown and Pittsfield stations, which stations, as I have before noticed, are presumed to be affected by a deflection of the plumb line,) with Bessel's latest mean result from the ten trigonometric surveys hitherto executed. For this purpose, I first ascertained that the meridians of the Massachusetts survey were sufficiently accurate, since a change in their convergency of $-2''.94$, in latitude $42^{\circ} 21' 30''$, for a difference of longitude of $1^{\circ} 58' 30''.27$, would fit them to Bessel's elements. I then proceeded to compute the value of the degree perpendicular to the meridian for the latitude of the State House at Boston, (namely, $42^{\circ} 21' 30''$,) by means of Mr. Paine's longitudes of the principal stations, which were obtained directly from the State House, and omitting those intermediate stations which were not directly compared therewith, excepting Nantucket, which, by Mr. Paine's report, appears to have been compared directly with New Bedford. But, as I have presumed there might exist a difference of opinion respecting Nantucket, that is, whether it should be compared with Boston or New Bedford, I have compared it with both, and have taken a mean of the results; presuming, further, that Mr. Paine frequently hastened his journey as rapidly as possible from Boston to Nantucket, and from Nantucket to Boston, by the way of New Bedford, and that therefore both comparisons would be equally entitled to consideration. Throughout the whole of these comparisons a relative weight, proportionate to the polar angle subtended, has been given to each value obtained.

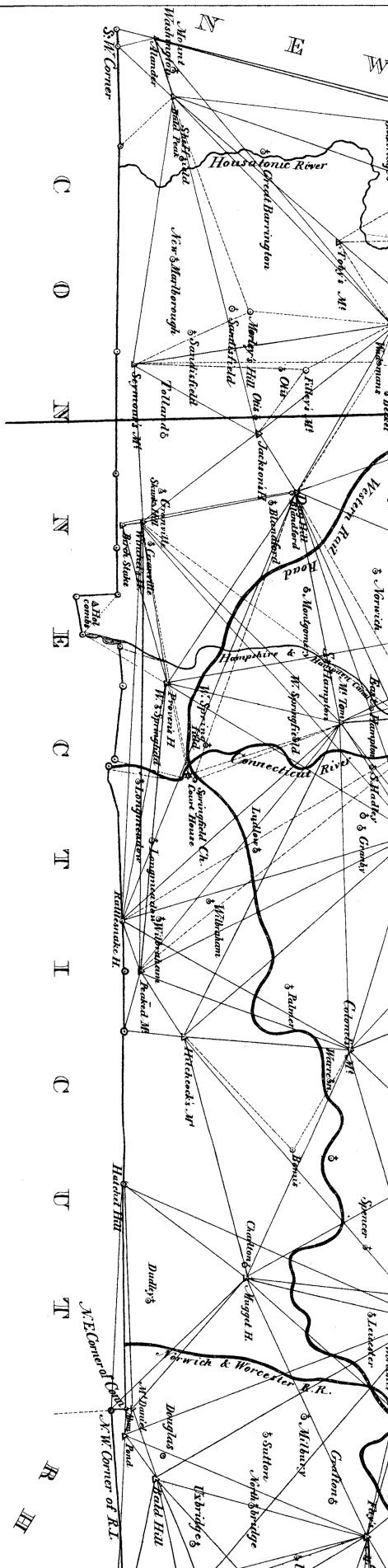
No.	Stations compared.	Degree perpendicular to the Meridian in feet.	Relative Weight.	Product in Feet.
		D.	W.	D \times W.
1	Boston and Northampton, . . .	365119	1.00	365119
2	" " Springfield, . . .	365356	0.97	354395
3	" " Greenfield, . . .	365198	0.98	357894
4	" " Worcester, . . .	365904	0.47	171976
5	" " Barnstable, . . .	365804	0.48	175586
6	" " Gloucester, . . .	365991	0.25	91498
7	" " Monomoy, . . .	366382	0.68	249140
8	" " Plymouth, . . .	367653	0.25	91913
9	" " Truro, . . .	366897	0.64	234814
10	" " Nantucket, . . .	365329	0.62	226504
		Aggregate,	6.34	2318839
		Paine's mean value,		365747

By comparing the Nantucket station with New Bedford, the tenth result becomes 364690 feet, with a weight of 0.52. This, substituted for No. 10 in the table, gives a mean result 365701, which, added to the result before obtained, and divided by 2, gives 365724 feet for the value of the perpendicular degree. This value I have adopted in computing the succeeding elements, which are compared, in the following table, with corresponding elements obtained by Bessel.

ELEMENTS.	Values from Bessel's Elements.	Borden's Values from the Massachusetts Survey.	Discrepancy.
Equatorial radius in English feet,	20923597.14	20921935.54	
“ “ “ “ miles.	3962.803	3962.487	0.316 miles
Polar semi-axis in English feet.	20853657.16	20850374.32	
“ “ “ “ miles.	3949.556	3948.935	0.621 miles
Degree of the meridian for latitude $42^{\circ} 21' 30''$ in feet,	364403.28	364356.00	47.28 feet
Degree perpendicular to meridian for do. do. do.	365740.21	365724.00	16.21 feet
Degree of the parallel for latitude $42^{\circ} 21' 30''$ do.	270262.09	270250.10	12.00 feet
Ellipticity,	$\frac{1}{299}$	$\frac{1}{292}$	$\frac{1}{007}$

The conclusion from this examination is, that chronometric comparisons do afford the means of determining one of the elements of the reduction of a trigonometric survey, namely, the value of a degree perpendicular to the meridian, and that, too, with a degree of uniformity quite too great to be the result of a happy accident.

NOTE BY THE COMMITTEE.—Mr. Boutelle informs the Committee that Rittenhouse, in tracing the boundary between Massachusetts and New York, had determined his course in such a manner as to agree with that of the Massachusetts survey within 7", and that his latitude differed from theirs only 5".



A

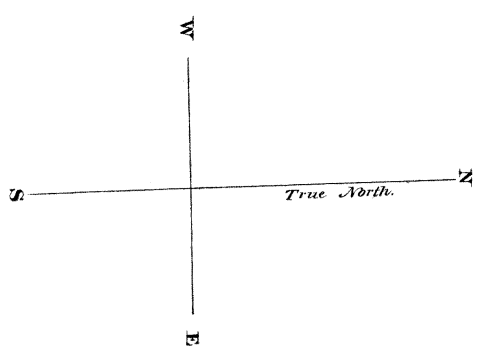
PLAN

OF THE PRINCIPAL TRIANGLES IN THE TRIGONOMETRICAL SURVEY OF MASSACHUSETTS.

Explanation.

- Primary Stations (except in the State Line) are marked with a triangle, thus Δ
- Secondary Stations, with a circle, \circ
- and, if Churches, a cross is added \times
- Astronomical Stations are designated by a star \star
- a star \star
- \star \star \star

Lith. of Sinclair, Philad.



Scale
Ten Miles to an Inch.

Engr'd on Stone by A. Francis.

